Math 210 — Practice Test 2

(1) Give an example three vectors in $\mathbb{R}^3$ for which no two are parallel and yet all three span a plane. Do they form a basis for that plane?

(2) If $A\mathbf{x} = \mathbf{0}$ has just one solution, then what's the most you can say about the matrix A? What if A were a square matrix?

(3) If the row vectors of an $n \times n$ matrix A are each perpendicular to a non-zero vector $\mathbf{v}$, then A cannot be invertible. Why not?

(4) Draw the Row Picture and Column Picture that illustrate the equation $\begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and its solution.

(5) (a) Draw both the schematic and "literal" pictures of the four fundamental subspaces associated to the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}$.

(b) On the left side (the row picture) of both the schematic and literal pictures, what corresponds to the set of solutions to $A\mathbf{x} = \mathbf{0}$?

(c) Find the complete solution $\mathbf{x} = \mathbf{x_p} + \mathbf{x_n}$ to the equation $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} \begin{bmatrix} 5 \\ 10 \end{bmatrix}$. Draw the set of solutions in the left side (the row picture) of the literal picture. Is this set of solutions a subspace?

(d) Repeat this problem using the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

(6) (a) Project the vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ into the line W spanned by $\mathbf{w} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$.

(b) Determine the matrix P that projects vectors into W.

(c) If a vector $\mathbf{x}$ is in $W^{\perp}$, why must $P\mathbf{x} = \mathbf{0}$?

(d) Use this to find a basis for $W^{\perp}$.

(7) (a) Find the complete solution $\mathbf{x} = \mathbf{x_p} + \mathbf{x_n}$ to the equation $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & -2 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 & 2 \\ 2 & 0 & 0 & 3 & -6 \\ 2 & -4 & 2 & 0 & -6 \end{bmatrix}$ and $\mathbf{b} \begin{bmatrix} -1 \\ 1 \\ 0 \\ -2 \end{bmatrix}$.

(b) Find bases for each $C(A)$, $C(A^T)$, and $N(A)$.

(8) These questions are about the vector space $\mathbf{M}$ of 2×2 matrices (where the "vectors" are 2×2 matrices).

(a) Give a basis for this vector space $\mathbf{M}$ and explain why it is a basis.

(b) What is the dimension of $\mathbf{M}$?

(c) Do the 2×2 matrices A such that $A^{\mathrm{T}} = -A$ form a vector space? If so, give a basis and state its dimension. If not, give a reason why not.

(d) Do the invertible 2×2 matrices form a vector space? If so, give a basis and state its dimension. If not, give a reason why not.