Math 210 — Practice Test 1

(1) Give an example of two square matrices A and B such that $(AB)^2 \neq A^2B^2$.

(2) Find an example of two non-square matrices A and B such that $AB = I_{2\times 2}$, the 2×2 identity matrix.

(3) For non-zero vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$: If $\mathbf{u}$ is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$, and $\mathbf{v}$ is perpendicular to $\mathbf{w}$, then can $\mathbf{u}$ be a linear combination of $\mathbf{v}$ and $\mathbf{w}$? Why or why not?

(4) If the row vectors of an $n \times n$ matrix A are each perpendicular to a non-zero vector $\mathbf{v}$, then A cannot be invertible. Why not?

(5) The column vectors $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ can also be viewed as 3×1 matrices.

(a) Check that their dot product can be viewed as a matrix multiplication: $\mathbf{v} \bullet \mathbf{w} = \mathbf{v}^T\mathbf{w}$. (Of course the left side is a number while the right side is a 1×1 matrix, so we're only slightly cheating.)

(b) Also calculate the matrices $\mathbf{v}\mathbf{w}^T$ and $\mathbf{w}\mathbf{v}^T$. How are they related? Are they invertible?

(6) Draw the Row Picture and Column Picture that illustrate the equation $\begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}\mathbf{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and its solution.

(7) (a) Use Elimination to make the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 2 & 4 \\ 6 & 0 & -3 \end{bmatrix}$ into an Upper Triangular matrix U. This can be done in three steps.

(b) Write out the corresponding elimination matrices M_1, M_2, M_3 that cause this transformation: $M_3M_2M_1A = U$.

(c) Do a full Gauss-Jordan Elimination on A to obtain A^{-1}.

(d) What is the matrix $(A^T)^{-1}$?