Practice Problems for Final

(A) Review previous Practice Problems.
(B) Additional Problems From the Book:

- p757 Concept Check 7, 8, 9
- p758–759 Exercises 7, 8, 12, 23–27, 29-34, 42–47, 49, 13–17, 21, 22
- p836 Concept Check 1-16 (could skip 3c)
- p837 1–14, 16, 17, 25, 26, 27, 29, 37, 38

Extras:

(1) Find both (a) a normal vector and (b) an equation for the plane tangent to the surface that is the graph of the function $f(x,y) = 3x^2 - y^2 + 2x$ at the point $(1, -2, f(1, -2))$.

(2) The position of a particle over time t is given by the vector function $\mathbf{r}(t) = \langle t^2 - 1, t^2 + 1, t^3 \rangle$. Find its (i) velocity, (ii) speed, and (iii) direction (i.e. the unit tangent vector $\mathbf{T}$) as functions of time t. (iv) Give an equation for the line tangent to the curve $\mathbf{r}(t)$ at time $t = 1$.

(3) Evaluate the integral $\displaystyle\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} \sqrt{x^2+y^2}\, dz\, dy\, dx$ by changing to cylindrical coordinates.

(4) Let S be the solid bounded by the parabolic cylinder $x = y^2$ and the planes $x = z$, $z = 0$, and $x = 1$.
  (a) Find the volume of S.
  (b) Find the x-coordinate of the centroid of S.

(5) Set up an iterated integral to calculate the volume of the solid in region $x \geq 0$, $y \geq 0$, and $z \geq 0$ beneath the cone $z = \sqrt{x^2 + y^2}$ and within the sphere $x^2 + y^2 + z^2 = 9$ using spherical coordinates.

(6) (‡ See edit ‡) Let's generalize the volume of the sphere of radius a.
  (a) Use a double integral to calculate the Area of the circle $x^2 + y^2 = a^2$.
  (b) Use a triple integral to calculate the Volume of the sphere $x^2 + y^2 + z^2 = a^2$.
  (c) Use a quadruple integral to calculate the "HyperVolume" of the "hypersphere" $x^2 + y^2 + z^2 + w^2 = a^2$.
  (d) Find a formula for the "n–volume" of the n–Sphere $x_1^2 + x_2^2 + \cdots + x_n^2 = a^2$? (No, this won't be on the final.)

Edit: Polar and Spherical coordinates make (a) and (b) simple. Using Euclidean coordinates requires
$\int 2\sqrt{a^2 - u^2}\, du = u\sqrt{a^2 - u^2} + a^2 \arcsin(u/a) + C$
which you do not need to memorize for the exam. The generalizations (c) and (d) are more challenging.

(7) Consider the vector field $\mathbf{F}(x,y,z) = \langle yz, xz, xy + z^2 \rangle$. Let C be the arc of the intersection of the plane $x = y$ with the sphere $x^2 + y^2 + z^2 = 9$ from $(0,0,-3)$ to $(0,0,3)$ that goes through the point $(3\sqrt{2}/2, 3\sqrt{2}/2, 0)$.
  (a) Find a function f so that $\mathbf{F} = \nabla f$.
  (b) Compute the work done by $\mathbf{F}$ along C.

(8) Evaluate the integral $\displaystyle\int_C (xe^z + ze^x) ds$ where C is the line segment from $(0,0,0)$ to $(3,0,3)$.

(9) Use Green's Theorem to evaluate the line integral $\displaystyle\int_C (y + e^{\sqrt{x}})dx + (2x + \cos y^2)dy$ counter-clockwise around the curve C that is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

(10) Use the Divergence Theorem to calculate the surface integral $\displaystyle\iint_S \mathbf{F} \cdot \mathbf{n} dS$ where $\mathbf{F}(x,y,z) = \langle 3xy^2, xe^z, z^3 \rangle$ where S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.