Practice Problems for Exam 2

From the Book:

- p691 Concept Check 7, 12 – 16
- p692–693 Exercises 5, 8, 17, 18, 23, 26, 29, 33, 41, 43–50
- p757 Concept Check 3
- p758–759 Exercises 3–6, 9–11, 13–17, 21, 22

Extras:

(1) The equation $t = \ln(r + st)$ implicitly defines $t$ as a function of $r$ and $s$. Find both $\partial t/\partial r$ and $\partial t/\partial s$.

(2) Let $f(x, y) = 7 - 3x^2 - 6y^2$.
  (a) Find the gradient of $f$ at the point $(-1, 1)$.
  (b) For the level curve containing the point $(-1, 1)$, find an equation for the tangent line to this level curve at the point $(-1, 1)$.
  (c) Sketch the level curve through the point $(-1, 1)$, the tangent line at this point, and the gradient vector at this point.

(3) Consider the function $f(x, y) = e^{1-x} - 2xy^3$.
  (a) Find the rate of change of $f$ at $(1, 2)$ in the direction of $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$.
  (b) In what direction (unit vector) does $f$ decrease at the maximum rate? What is this maximum rate of change?
  (c) In what directions is the rate of change of $f$ at $(1, 2)$ equal to 0? Your answer should be a pair of opposite unit vectors.

(4) Suppose the gradient $\nabla f(2, 4)$ of a function $f(x, y)$ has length equal to 5. Is there a direction $\mathbf{u}$ such that the directional derivative $D_{\mathbf{u}}f$ at the point $(2, 4)$ is 7? Explain your answer.

(5) Find the tangent plane to the surface $x^2 - 4y^2 + 9 = z^2$ at the point $P = (2, 1, 3)$.

(6) Find the points on the surface $x^2 + 2y^2 - 4z^2 + 2xy + 8xz = 4$ where the tangent plane is parallel to the $xz$–plane.

(7) Find all the critical points of $f(x, y) = x^3 + xy^2/2 + x^2y$ and apply the second derivative test to each of them.

(8) Find the absolute maximum and minimum values of the function $f(x, y) = (x-1)^2+(y-1)^2$ in the rectangular domain $D = \{(x, y)\colon 0 \le x \le 1, 0 \le y \le 2\}$. Justify your answer.

(9) Compute $\iint_D (3x + 1)\ dx\,dy$ where $D$ is the region in the first quadrant bounded by the parabolas $y = x^2$ and $y = (x - 1)^2$ and the $y$–axis.

(10) Change the order of integration in the following iterated integral:

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}} f(x, y)\ dx\,dy.$$

(11) (a) Find the area of the region enclosed by the cardioid given in polar coordinates by $r = 1 + \cos(\theta)$.
  (b) Use polar coordinates to evaluate

$$\int_{0}^{\sqrt{2}}\int_{y}^{\sqrt{4-y^2}} \frac{1}{1 + x^2 + y^2}\ dx\,dy.$$

(12) Find the centroid of a quarter of a unit disk.