Hwk 1 MTH533

1. Show that

(a) $(-1)x = -x$

(b) $-(-x) = x$

(c) $(-1)(-1) = 1$

(d) $0 \cdot x = 0$

(e) $x > y \Rightarrow -x < -y$

(f) $-x$ is unique

2. Describe exactly all steps you use to solve the equation

$$ax + b = c$$ in a

Commutative field (like $\mathbb{R}$).

3. Can $\underbrace{x + x + \cdots + x}_{n \text{ times}} = 0$?

4. If $\frac{a}{b}$, $b \neq 0$ means ab^{-1}

then $\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$.

(5) Show that a finite field cannot be ordered, i.e., there is no finite field satisfying the compatibility axioms with the order relation.

<u>Hint</u> Add $\underbrace{1+1+\cdots+1}_{n}$ for various n and draw a contradiction.

(6) $Q[\sqrt{2}] = \{ p+\sqrt{2}q \mid p, q \in Q \}$

Show that $Q[\sqrt{2}]$ is a commutative field. Assume $\sqrt{2}$ is known to be irrational.

What is p, q in $\dfrac{3-\sqrt{2}}{-1+\sqrt{2}} = p + q\sqrt{2}$?

(7) Show that there exists only one $x > 0$ such that $x^n = 2$, $n \geq 1$ integer.

(8) Show $a>0$ $a^{\frac{x}{y}} = (a^x)^{\frac{1}{y}} = (a^{\frac{1}{y}})^x$ where $x, y \in N$

(9) Show that $\mathbb{Q} \times \mathbb{Q}$ with the lexicographic order is an ordered set,

$(p_1, q_1) < (p_2, q_2)$ if

$(p_1 < p_2)$ or $(p_1 = p_2$ and $q_1 < q_2)$.

(10) Show that $\mathbb{Q} \times \mathbb{Q}$ with

$(p_1, q_1) \leq (p_2, q_2)$

if $p_1 \leq p_2$ and $q_1 \leq q_2$

is an ordered set but not any pair of elements of $\mathbb{Q} \times \mathbb{Q}$ are comparable.

* Notice that they are in (9).