NOTES ON HOMEWORK 1

Problem A. We introduce the *lexicographic order* on $\mathbb{Z} \times \mathbb{Z}$. We say

$$(p_1, q_1) \leq (p_2, q_2) \quad \text{if either}$$

$$(p_1 < p_2) \qquad \text{or} \qquad (p_1 = p_2 \quad \text{and} \quad q_1 \leq q_2)\,.$$

Here $\leq$ means *less or equal* and $<$ means *strictly less*, in the usual sense.

(i) Show that the lexicographic relation is indeed an order relation.

(ii) Show that *any two pairs can be compared, i.e. we can determine which is less than the other*. We then say that the set $\mathbb{Z} \times \mathbb{Z}$ is **totally ordered**.

(iii) Sketch how to generalize this relation for $\mathbb{Z}^k$, $k \geq 2$? What if the product is over infinitely many copies of $\mathbb{Z}$ (infinite words)?

Problem B. We introduce the *coordinatewise order* on $\mathbb{Z} \times \mathbb{Z}$. We say

$$(p_1, q_1) \leq (p_2, q_2) \quad \text{if}$$

$$(p_1 \leq p_2) \qquad \text{and} \qquad (q_1 = q_2)\,.$$

(i) Show that this relation is indeed an order relation.

(ii) Show that *NOT any two pairs can be compared*. We say that the set $\mathbb{Z} \times \mathbb{Z}$ is **partially ordered**.