EQUIVALENT DEFINITIONS OF THE SUPREMUM

If A is bounded above, then $\sup A$ is the *lowest upper bound* of A.

Equivalent definition:
$\sup A$ must satisfy both conditions

- $\forall x \in A$ we have $x \leq \sup A$
- $\forall \epsilon > 0\ \exists x \in A$ such that $x > \sup A - \epsilon$

The first condition says that $\sup A$ is an upper bound of A

The second condition says that no number less than $\sup A$ is an upper bound for A. If we pick a number smaller than A even by a small amount, that is $\sup A - \epsilon$, there must be an x larger than it, otherwise $\sup A - \epsilon$ would be an upper bound for all $x \in A$, and $\sup A > \sup A - \epsilon$ would have not been the lowest upper bound.