Hwk 2.1 – 2.2

2.1

2 (a) $-(a+b) = (-a) + (-b)$

Show that if we add $a+b$ to $(-a)+(-b)$ we obtain 0. Then $(-a)+(-b)$ is the unique opposite [inverse wrt $+$] of $a+b$.

proof $\quad [(-a)+(-b)] + [a+b]$

$= [(-a) + a] + [(-b)+b] = 0 + 0 = 0$

using associativity to change the order in the addition, commutativity and finally the fact that $-a$ is the inverse of a, and $-b$ — " — b.

Second proof $\quad -(a+b) = (-1)(a+b)$

(you need to show 1 (c), done in class)

but $-a = (-1)a$; $-b = (-1)b$ and so.

$(-1)(a+b) = (-1)a + (-1)b$ by DISTRIBUTIVITY

we are done.

Remark In these proofs, consistency is essential (What you know first, vs. what to deduce)

2.1 2(b) similar to 2.1. 2(a)

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9 The only "harder" part is

$$\frac{s_1 + t_1\sqrt{2}}{s_2 + t_2\sqrt{2}} = s + t\sqrt{2} \quad \text{for some } s, t \in \mathbb{Q}.$$

proof multiply the denominator by $s_2 - t_2\sqrt{2}$ [Rationalization]

$$\Rightarrow \frac{s_1 + t_1\sqrt{2}}{s_2 + t_2\sqrt{2}} = \frac{(s_1 + t_1\sqrt{2})(s_2 - t_2\sqrt{2})}{s_2^2 - 2t_2^2} =$$

$$= \underbrace{\left(\frac{s_1 s_2 - 2t_1 t_2}{s_2^2 - 2t_2^2}\right)}_{\text{new "s"}} + \underbrace{\left(\frac{t_1 s_2 - s_1 t_2}{s_2^2 - 2t_2^2}\right)}_{\text{new "t"}} \sqrt{2}.$$

done The rest is easy / factor $\sqrt{2}$ when necessary. Provide a few details.

2.1 (23) This is $\Leftrightarrow$ if AND ONLY if

$0 < a < b \Rightarrow a^n < b^n.$

$a = b \Rightarrow a^n = b^n.$

Induction True for $n = 1$.

$P(n) \Rightarrow P(n+1)$.

$$a^{n+1} = \underbrace{a \cdot a^n < b}\cdot a^n < b \cdot b^n$$

use $A < B$; $C > 0$ $AC < BC$

with $A = a$ $B = b$; $C = a^n$

then $A = a^n$ $B = b^n$; $C = b$ here we use $P(n)$!

The equality statement is obvious.

$\boxed{0 < a < b}$

Reciprocal $a^n < b^n \Rightarrow a < b$.

If $a^n < b^n$ but $a \geq b$, then

either. 1) $a = b \Rightarrow a^n = b^n$ false

2) $a > b \Rightarrow a^n > b^n$ false

We proved the statement by contradiction.

Think of proof based on Algebra.

2.2 10 (a). $|x-1| > |x+1|$

we have to analyze. $x-1 \geq 0$; < 0

$x+1 \geq 0$; < 0

case by case.

(i) $x-1 \geq 0$; $x+1 \geq 0$

$x-1 > x+1$ impossible.

(ii) $x-1 \geq 0$; $x+1 < 0$

$x-1 > -x-1 \iff 2x > 0 \iff x > 0.$

But $x \not\geq 1$. $[1,\infty)$

$x < -1$ $(-\infty, -1)$ intersection ϕ

$x > 0$ $(0,\infty)$

(iii) $x-1 < 0$; $x+1 \geq 0$.

$-x+1 > x+1$ $0 > 2x$

$x < 0$. we have. $(-\infty, 1) \cap [-1,\infty) \cap$

$(-\infty, 0) = [-1, 0)$

(iv) $x-1 < 0$; $x+1 < 0$

$-x+1 > -x-1$ Always true.

$(-\infty, 1) \cap (-\infty, -1) = (-\infty, -1)$.

Answer $(-\infty,-1) \cup [-1,0) = (-\infty, 0)$ ✓

[Alternative solution: square. Warning — does not work all the time]

5

2.2 18 simply verify.

(a) if $a > b$ $\max\{a, b\} = a$.

$\frac{1}{2}(a+b+|a-b|) = \frac{1}{2}(a+b+a-b) = a$.

if $a < b$ --- same reasoning. Answer b.

if $a = b$ then either is correct.

In all three cases there is equality

$\Rightarrow$ by trichotomy, the formula must be correct.

(b) $\min\{a, b, c\} = \min\{\min\{a, b\}, c\}$.

Let's compare c with $m = \min\{a, b\}$.

b1 If $c \geq m$ then LHS $= m$

because it means $c \geq a$ or $c \geq b$

so $\min\{a, b, c\} = \min\{a, b\} = m$.

b2 If $c < m$ then $c < a$ and $c < b$

$\Rightarrow$ LHS $= c$

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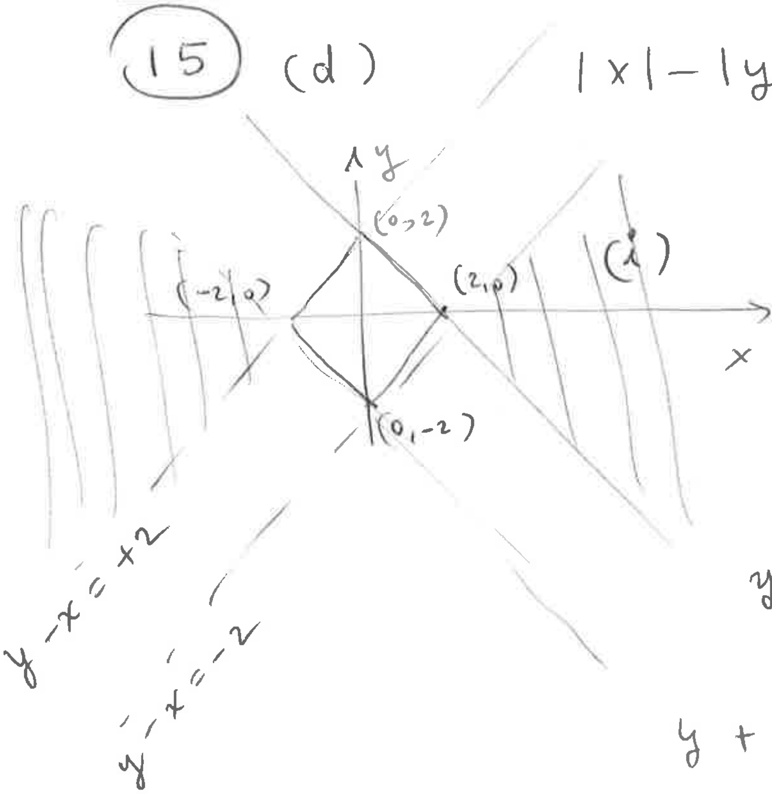
b1 RHS is m. ✓

b2 RHS is c ✓

Both are = to LHS.

$\frac{2,2}{15}$ (d) $|x|-|y| \geq 2$.

(6)



✓ Correct region ////

$$R = \{(x,y) \mid y>0,\ y \leq |x|+2\}$$

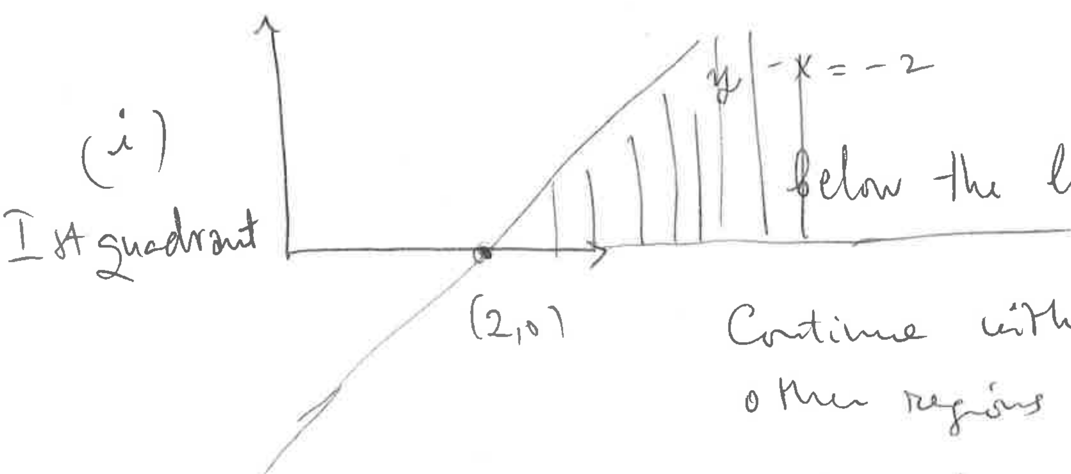
$$\cup \{(x,y) \mid y<0,\ -y \leq |x|+2\}$$

We split in 4 cases:

$x \geq 0$	$y \geq 0$	(i)
$x \geq 0$	$y < 0$	(ii)
$x < 0$	$y \geq 0$	(iii)
$x < 0$	$y < 0$	(iv)

Explicitly solve in each. Take the union set of solutions

If (i). $x - y \geq 2 \Rightarrow y - x \leq -2$.

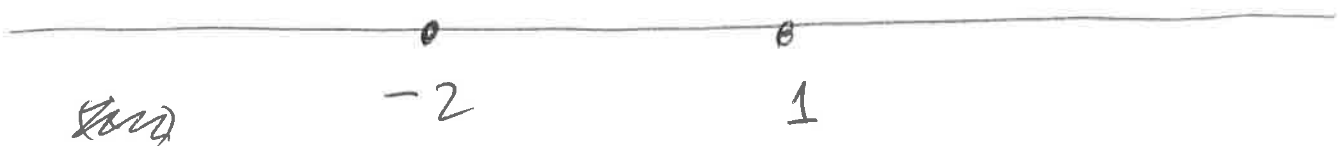


Continue with the other regions.

Alt. sol.: in class

2.2. 12 find $x \in \mathbb{R}$ s.t.

$$4 < |x+2| + |x-1| < 5.$$



$x \in [1, \infty)$: $4 < x+2+x-1 < 5$ $\quad 3 < 2x < 4$

$\frac{3}{2} < x < 2 \Rightarrow x \in \left(\frac{3}{2}, 2\right)$

$x \in [-2, 1)$: $4 < x+2-x+1 < 5$ $\quad 4 < 1 < 5$

impossible. $x \in \emptyset$

$x \in (-\infty, -2)$: $4 < -x-2-x+1 < 5$.

$$5 < -2x < 6$$

$$-3 < x < -\frac{5}{2}$$

$$x \in \left(-3, -\frac{5}{2}\right).$$

Answer $\left(-3, -\frac{5}{2}\right) \cup \left(\frac{3}{2}, 2\right)$.