MTH 230

Ch 3

Ex 52

52 modified

62

Ch 3, Ex 52

Solve

$2x \equiv 11 \pmod{13}$

$3x \equiv 7 \pmod{9}$

$7x \equiv 5 \pmod{8}$

This equation does not have solutions.

The reason is eq. #2:

$\gcd(3,9) = 3$ and $3 \nmid 7$.

So the second equation does not have solutions.

We did this in class. However, let's modify equation (2) to have a system with solutions, to illustrate how to solve it. See next page.

pb 3.52 $\longmapsto$ modified

$$\begin{cases} 2x \equiv 11 \pmod{13} \\ 3x \equiv 6 \pmod{9} \\ 7x \equiv 5 \pmod{8}. \end{cases}$$

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we have. $7 \cdot 2 = 14 \Rightarrow \hat{7} \cdot \hat{2} = \widehat{14} = \hat{1}$

in $\mathbb{Z}_{13}$ or $\hat{7} = \hat{2}^{-1}$ in $\mathbb{Z}_{13}$.

$\hat{2}\hat{x} = \widehat{11} \Rightarrow \hat{7} \cdot \hat{2} \cdot \hat{x} = \hat{7} \cdot \widehat{11} \Rightarrow$

$\Rightarrow \widehat{14}\,\hat{x} = \widehat{77} \Rightarrow \hat{1} \cdot \hat{x} = \widehat{65+12} \Rightarrow \hat{x} = \widehat{12}$

$\Rightarrow \boxed{x \equiv 12 \pmod{13}} \Rightarrow x = 12 + 13y$

Let's look at the second equation.

$3(12+13y) \equiv 6 \pmod{9} \Rightarrow \widehat{36} + \widehat{39}\,\hat{y} = \hat{6}$

in $\mathbb{Z}_9 \Rightarrow \hat{3}\,\hat{y} = \hat{6} \Rightarrow 9 \mid 3(y-2) \Rightarrow$

$\Rightarrow 3 \mid y-2 \Rightarrow \boxed{y = 2 + 3z}$

pb 3.52 (modified)

Third equation.

First, we now have

$x = 12 + 13y \qquad y = 2 + 3z \Rightarrow$

$$\boxed{x = 12 + 13(2+3z) = 12 + 26 + 39z = 38 + 39z}$$

plug into the 3^{rd} eq:

$$7(38 + 39z) \equiv 5 \pmod 8$$

$$\hat{7} \cdot \hat{38} + \hat{7} \cdot \hat{39}\,\hat{z} \equiv \hat{5} \quad \text{in } \mathbb{Z}_8$$

$$\hat{7} \cdot \hat{6} + \hat{7} \cdot \hat{7}\,\hat{z} = \hat{5}$$

$$\hat{z} = \hat{5} - \hat{42} = \widehat{-37} = \hat{3} \ (\mathbb{Z}_8)$$

$$z = 3 + 8w$$

Finally $\quad x = 38 + 39(3 + 8w)$

$$x = [38 + 3 \cdot 39] + 312w$$

$$\boxed{x = 155 + 312w}$$

verify!

$$x = 155 \pmod{312} \longleftarrow \text{Answer}$$

problem 3.62 ; The answer will be k odd

Let's find all $k \geq 1$ such that

$$\forall n \in \mathbb{Z} \qquad n^k \equiv n \pmod 6$$

Notice that $\forall\, k \geq 1$ we have

$2 \mid n^k - n$. This is because

$\hat{n}^k = \hat{n}$ for any $\hat{n} \in \mathbb{Z}_2 = \{\hat{0}, \hat{1}\}$.
(even/odd).

The problem is: for what $k \geq 1$

$3 \mid n^k - n$? So $\hat{n}^k = \hat{n}$, $\hat{n} \in \mathbb{Z}_3$

$\hat{n} \in \{\hat{0}, \hat{1}, \hat{2}\}$. $\qquad \hat{n} = \hat{0}$ true

$\hat{n} = \hat{1}$ true.

Answer

$\hat{n} = \hat{2} = \widehat{(-1)} \Rightarrow \widehat{(-1)}^k = \widehat{(-1)}$ true when k odd

But when k = even $\hat{1} \equiv \widehat{-1}$ in $\mathbb{Z}_3 \iff 3 \mid 2$ false