

problem 95. Ch 2

Why is  $\sum_{k=1}^n \frac{1}{k}$ ,  $n \geq 2$

NOT equal to an integer?

The reason is that we shall show

$$S_n = \sum_{k=1}^n \frac{1}{k} = \frac{p}{q} \text{ where } \begin{matrix} p \text{ odd} \\ q \text{ even} \end{matrix}$$

when  $n \geq 2$

=> cannot be simplified to an integer.

proof  $n=2 \Rightarrow \frac{3}{2} \checkmark$

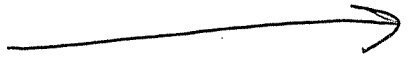
$n=3 \quad \frac{3}{2} + \frac{1}{3} = \frac{11}{6} \text{ true } \checkmark$

Suppose

$$S_n = \frac{p}{2^l m}, \quad \begin{matrix} p, m \text{ odd} \\ l \geq 1 \end{matrix}$$

Case I  $n+1$  even

Case II  $n+1$  odd



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Case I  $n+1 = \text{even} = 2^d r$ ,  $r$  odd.

$$\text{Then } \frac{p}{2^l m} + \frac{1}{2^d r} = \frac{2^d p r + 2^l m}{2^{l+d} m r}$$

if  $d \geq l$  we simplify

$$\frac{2^{d-l} p r + m}{2^d m r} = \frac{\text{odd}}{\text{even}} \text{ because } d \geq 1.$$

if  $l > d$  we simplify

$$\frac{p r + 2^{l-d} m}{2^l m r} = \frac{\text{odd}}{\text{even}}, \text{ since } l \geq 1,$$

Case II  $n+1 = \text{odd} \Rightarrow$

$$\frac{p}{2^l m} + \frac{1}{n+1} = \frac{p(n+1) + 2^l m}{2^l m (n+1)} = \frac{\text{odd}}{\text{even}}.$$

Since  $l \geq 1$

In all cases the fraction cannot be simplified  
 $\Rightarrow S_n \notin \mathbb{Z}$ . ✓