problem 88 Ch. 2

We want to know how many positive $x, y, z \in \mathbb{Z}$ there are such that

$$6x + 10y + 15z = 167$$

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we shall factor out $\gcd(10,15) = 5$

and solve

$$6x + 5\underbrace{(2y + 3z)}_{w} = 167$$

$$6x + 5w = 167.$$

Suppose we solved it. then for any solution (x_n, w_n) we solve $2y + 3z = w_n$

That's the plan.

$6 \cdot (1) + 5(-1) = 1 \Rightarrow (x_0, w_0) = (167, -167)$

$(x_n, w_n) = (167 + 5n, -167 - 6n)$

Solve $2y + 3z = w_n = -167 - 6n$

Since $2(-1) + 3(+1) = 1$

we obtain

$(y_0, z_0) = (-w_n, +w_n)$.

So for any m (NOT n again !)

$(y_m, z_m) = (-w_n + 3m, +w_n - 2m)$

Summarizing

$$(x, y, z) = (\overbrace{167+5n}^{x}, \overbrace{167+6n}^{y}+3m, \overbrace{-167-6n-2m}^{z})$$

It is natural that an equation with 3 unknowns would have two parameters. This happens in algebra because (x, y, z) are 3 (no equation no information)

$ax+by+cz=d$ three unkowns, one equation

i.e. 2 degrees of freedom

here $(m, n) \in \mathbb{Z} \times \mathbb{Z}$.

now if we want the ≥ 0 solutions we have to have

$$\begin{cases} 167+5n \geq 0 \\ 167+6n+3m \geq 0 \\ -167-6n-2m \geq 0 \end{cases}$$ last 2 $\Rightarrow 7m \geq 0$ by (+)

Clearly n cannot be + from the last eq.

$\boxed{m \geq 0,\ n \leq 0}$ $|n| = -n$.

$|n| \leq \frac{167}{5} = 33.4$ $\boxed{0 \leq |n| \leq 33}$

$3m - 6|n| \geq -167$

$m - 2|n| \geq -55.66$

$m - 2|n| \geq -55$

$6|n| - 2m \geq 167 \Rightarrow 3|n| - m \geq 83.5$

$3|n| - m \geq 84$

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we have

$2|n| - 55 \leq m \leq 3|n| - 84$

$\boxed{29 \leq |n|}$

$|n| =$ 29, 30, 31, 32, 33 try them all

any $29 \le |n| \le 33$ could lead to a solution.

$$2|n| - 55 \le m \le 84 - 3|n|$$

$$\Downarrow$$

$$5|n| \le 84 + 55 = 139$$

$$|n| \le 27.8$$

$\|n\|$	m
29	3
30	5, 6
31	7, 8, 9
32	9, 10, 11, 12
33	11, 12, 13, 14, 15

There are **15** ways to write 167¢ as the sum of 6¢, 15¢, 10¢.

Problem 88 Chapter 2

n	m	x	y	z	total
-29	3	22	2	1	167
-30	5	17	2	3	167
-30	6	17	5	1	167
-31	7	12	2	5	167
-31	8	12	5	3	167
-31	9	12	8	1	167
-32	9	7	2	7	167
-32	10	7	5	5	167
-32	11	7	8	3	167
-32	12	7	11	1	167
-33	11	2	2	9	167
-33	12	2	5	7	167
-33	13	2	8	5	167
-33	14	2	11	3	167
-33	15	2	14	1	167

x=167+5*n

y=167+6*n+3*m

z=-167-6*n-2*m