

HOMWORK 2

1. Let $U = (-\infty, \infty)$. Determine the truth values of the propositions. When they are false, write down the negation.

- a) $(\forall x)(2x = x^2)$
- b) $(\exists x)(2x = x^2)$
- c) $(\forall x)((x > -1) \rightarrow (x^2 > 1))$
- d) $(\exists x)((x > -1) \rightarrow (x^2 > 1))$

2. Express the statements using quantifiers and propositional calculus. Specify the appropriate universe of discourse U . Determine their truth value.

- a) *The difference between a positive number and a negative number is positive*
- b) *It is not always the case that if the product of two numbers is positive, then both are positive*
- c) *The ratio of two positive integers is always an integer*
- d) *The product of two integers is always a perfect square*

3. Let $U = \{\text{all integers}\}$. Determine the truth values of the propositions. When they are false, write down the negation.

- a) $(\forall n)(\forall m)(n^2 < m)$
- b) $(\exists m)(\forall n)(n^2 < m)$
- c) $(\exists m)(\exists n)(m^2 + n^2 = 5)$
- d) $(\exists m)(\exists n)(m^2 + n^2 = 6)$
- e) $(\forall n)(\forall m)(\exists p)(m + n = 2p)$

4. Let $U = (-\infty, \infty)$. Determine the truth values of the propositions. When they are false, write down the negation.

- a) $(\forall x)(\exists y)(x + y = 0)$
- b) $(\exists y)(\forall x)(x + y = 0)$
- c) $(\forall x)(\exists y)(xy = 1)$
- d) $(\forall x)(\exists y)((x \neq 0) \rightarrow (xy = 1))$