a) by showing each side is a subset of the other side.
b) using a membership table.

16. Let A and B be sets. Show that
a) $(A \cap B) \subseteq A$.
b) $A \subseteq (A \cup B)$.
c) $A - B \subseteq A$.
d) $A \cap (B - A) = \emptyset$.
e) $A \cup (B - A) = A \cup B$.

17. Show that if A, B, and C are sets, then $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$
a) by showing each side is a subset of the other side.
b) using a membership table.

18. Let A, B, and C be sets. Show that
a) $(A \cup B) \subseteq (A \cup B \cup C)$.
b) $(A \cap B \cap C) \subseteq (A \cap B)$.
c) $(A - B) - C \subseteq A - C$.
d) $(A - C) \cap (C - B) = \emptyset$.
e) $(B - A) \cup (C - A) = (B \cup C) - A$.

19. Show that if A and B are sets, then $A - B = A \cap \overline{B}$.

20. Show that if A and B are sets, then $(A \cap B) \cup (A \cap \overline{B}) = A$.

21. Prove the first associative law from Table 1 by showing that if A, B, and C are sets, then $A \cup (B \cup C) = (A \cup B) \cup C$.

22. Prove the second associative law from Table 1 by showing that if A, B, and C are sets, then $A \cap (B \cap C) = (A \cap B) \cap C$.

23. Prove the second distributive law from Table 1 by showing that if A, B, and C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

24. Let A, B, and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.

25. Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find
a) $A \cap B \cap C$.
b) $A \cup B \cup C$.
c) $(A \cup B) \cap C$.
d) $(A \cap B) \cup C$.

26. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.
a) $A \cap (B \cup C)$
b) $\overline{A} \cap \overline{B} \cap \overline{C}$
c) $(A - B) \cup (A - C) \cup (B - C)$

27. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.
a) $A \cap (B - C)$
b) $(A \cap B) \cup (A \cap C)$
c) $(A \cap \overline{B}) \cup (A \cap \overline{C})$

28. Draw the Venn diagrams for each of these combinations of the sets A, B, C, and D.
a) $(A \cap B) \cup (C \cap D)$
b) $\overline{A} \cup \overline{B} \cup \overline{C} \cup \overline{D}$
c) $A - (B \cap C \cap D)$

29. What can you say about the sets A and B if we know that
a) $A \cup B = A$?
b) $A \cap B = A$?
c) $A - B = A$?
d) $A \cap B = B \cap A$?
e) $A - B = B - A$?

30. Can you conclude that $A = B$ if A, B, and C are sets such that
a) $A \cup C = B \cup C$?
b) $A \cap C = B \cap C$?
c) $A \cup B = B \cup C$ and $A \cap C = B \cap C$?

(c) $A = \{y, z\}$ $B = \{u, y, z\}$ $C = \{z\}$

31. Let A and B be subsets of a universal set U. Show that $A \subseteq B$ if and only if $\overline{B} \subseteq \overline{A}$?

The **symmetric difference** of A and B, denoted by $A \oplus B$, is the set containing those elements in either A or B, but not in both A and B.

32. Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$.

33. Find the symmetric difference of the set of computer science majors at a school and the set of mathematics majors at this school.

34. Draw a Venn diagram for the symmetric difference of the sets A and B.

35. Show that $A \oplus B = (A \cup B) - (A \cap B)$.

36. Show that $A \oplus B = (A - B) \cup (B - A)$.

37. Show that if A is a subset of a universal set U, then
a) $A \oplus A = \emptyset$.
b) $A \oplus \emptyset = A$.
c) $A \oplus U = \overline{A}$.
d) $A \oplus \overline{A} = U$.

38. Show that if A and B are sets, then
a) $A \oplus B = B \oplus A$.
b) $(A \oplus B) \oplus B = A$.

39. What can you say about the sets A and B if $A \oplus B = A$?

*40. Determine whether the symmetric difference is associative; that is, if A, B, and C are sets, does it follow that $A \oplus (B \oplus C) = (A \oplus B) \oplus C$?

*41. Suppose that A, B, and C are sets such that $A \oplus C = B \oplus C$. Must it be the case that $A = B$?

42. If A, B, C, and D are sets, does it follow that $(A \oplus B) \oplus (C \oplus D) = (A \oplus C) \oplus (B \oplus D)$?

43. If A, B, C, and D are sets, does it follow that $(A \oplus B) \oplus (C \oplus D) = (A \oplus D) \oplus (B \oplus C)$?

*44. Show that if A, B, and C are sets, then

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

(This is a special case of the inclusion–exclusion principle, which will be studied in Chapter 7.)

*45. Let $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$. Find
a) $\bigcup_{i=1}^{n} A_i$.
b) $\bigcap_{i=1}^{n} A_i$.

*46. Let $A_i = \{\ldots, -2, -1, 0, 1, \ldots, i\}$. Find
a) $\bigcup_{i=1}^{n} A_i$.
b) $\bigcap_{i=1}^{n} A_i$.

47. Let A_i be the set of all nonempty bit strings (that is, bit strings of length at least one) of length not exceeding i. Find
a) $\bigcup_{i=1}^{n} A_i$.
b) $\bigcap_{i=1}^{n} A_i$.

48. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,
a) $A_i = \{i, i + 1, i + 2, \ldots\}$.
b) $A_i = \{0, i\}$.
c) $A_i = (0, i)$, that is, the set of real numbers x with $0 < x < i$.
d) $A_i = (i, \infty)$, that is, the set of real numbers x with $x > i$.