Divisibility criteria

How do we tell if n is divisible by another number?

Assume n is written in base $\underline{10}$

- by 2 : last digit even
  4 : —"— 2 $\vdots$ 4
- by 3 : sum of digits $\vdots$ 3
  9 : $\vdots$ 9
- by 5 : last digit $\vdots$ 5 i.e. 0, 5
- by 6 : by 2 & by 3
- by 10 : must have a zero at the end.
- by 11 : alternating sum of digits $\vdots$ 11.

Since $n = (d_k d_{k-1} \cdots d_1 d_0)_{10}$

i.e. $d_k \cdot 10^k + d_{k-1} \cdot 10^{k-1} + \cdots + d_1 \cdot 10^1 + d_0 \cdot 10^0$

ex $k = 2$ $\quad 259 = 2 \cdot 10^2 + 5 \cdot 10^1 + 9 \cdot 10^0$

we have $n = 10 n_1 + d_0$.

Evidently divisibility by $2, 5, 10$ are given by d_0.

$2 \mid 10 n_1 + d_0 \iff 2 \mid d_0$.

since $10 = 2 \cdot 5$ $\qquad d_0 = 0, 2, 4, 6, 8$

$5 \mid 10 n_1 + d_0 \iff 5 \mid d_0$ i.e. $d_0 = 0, 5$.

how about 3 ?

notice that

$d_k 10^k + d_{k-1} 10^{k-1} + \cdots + d_1 \cdot 10 + d_0 =$

$= d_k[\underbrace{10^k - 1} + 1] + d_{k-1}[10^{k-1} - 1 + 1] + \cdots d_1[10 - 1 + 1] + d_0$

all $10^k - 1 = 9 \cdot \{\ldots\}$ so $\boxed{d_k + d_{k-1} + \cdots + d_0}$ determines if 3 or 9 divide $\underline{n}$.

How about 11 ?

write $10 = 11 - 1$. then.

$$10^k = (11-1)^k = \underline{\text{foil}} :$$

~~...~~ take $k = 3$ for example.

$$(11-1)^3 = 11^3 + 3 \cdot 11^2(-1) + 3 \cdot 11 \cdot (-1)^2 + (-1)^3$$

$$= 11 \cdot n_1 + (-1)^3 .$$

we then get

$$n = \sum_{i=0}^{k} d_i (11-1)^i = \text{multiple of } 11 +$$

$$+ \underbrace{\sum_{i=0}^{k} (-1)^i d_i}$$

the alternating sum of digits !