1.5-16 4 Lose, 1 Win chips

$$P\left(\text{the first player wins}\right) = P\left(\text{the white chip is drawn on an odd draw}\right)$$

$$= \sum_{\substack{x = 1,2,\ldots \\ \text{odd}}} f(x)$$

$$f(x) = P\left(\text{the white chip is drawn on draw \# } x\right)$$

$\boxed{\text{w/o replacement}}$ $f(x) = \frac{1}{5}$, $x = 1,2,3,4,5$

there are many ways to see that. Directly, it is

$$f(x) = P(A \cap B) = P(A \mid B)\, P(B)$$

$A = \{\text{the } x^{th} \text{ chip is } W\}$

$B = \{\text{there are exactly zero } W \text{ among the first } x-1 \text{ draws}\}$

$$f(x) = \frac{1}{5-(x-1)} \cdot \frac{\binom{1}{0}\binom{4}{x-1}}{\binom{5}{x-1}} = \frac{1}{5} \quad \left(\text{after the algebra}\right)$$

So $P\left(1^{st} \text{ player wins}\right) = f(1) + f(3) + f(5) = \boxed{\frac{3}{5}}$

Problem 1.5 – 16 continued

with replacement $\quad x = 1, 2, 3, \ldots$ any number is possible

$$P(A \cap B) = P(A \mid B)\, P(B)$$

$$\frac{1}{5} \cdot P\left(\begin{array}{c}\text{there are}\\ \text{exactly}\\ \text{zero W among}\\ \text{the first } x-1 \text{ draws}\end{array}\right)$$

$$f(x) = \underbrace{\binom{x-1}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{x-1}}_{\frac{1}{5}\cdot\left(\frac{4}{5}\right)^{x-1}}$$

denote $p = \frac{1}{5}$.

$$f(1) + f(3) + f(5) + \ldots$$

$$= \frac{1}{5}\left[\left(\frac{4}{5}\right)^{1-1} + \left(\frac{4}{5}\right)^{3-1} + \left(\frac{4}{5}\right)^{5-1} + \left(\frac{4}{5}\right)^{7-1} + \ldots\right]$$

$$= p\left[1 + (1-p)^2 + (1-p)^{2\cdot 2} + (1-p)^{2\cdot 3} + \ldots\right]$$

$$= p\left[\frac{1}{1-(1-p)^2}\right] = \frac{p}{(1-1+p)(1+1-p)} = \frac{1}{2-p} = \frac{5}{9}$$

We used $1 + z + z^2 + \ldots = \frac{1}{1-z}$ when $|z| < 1$