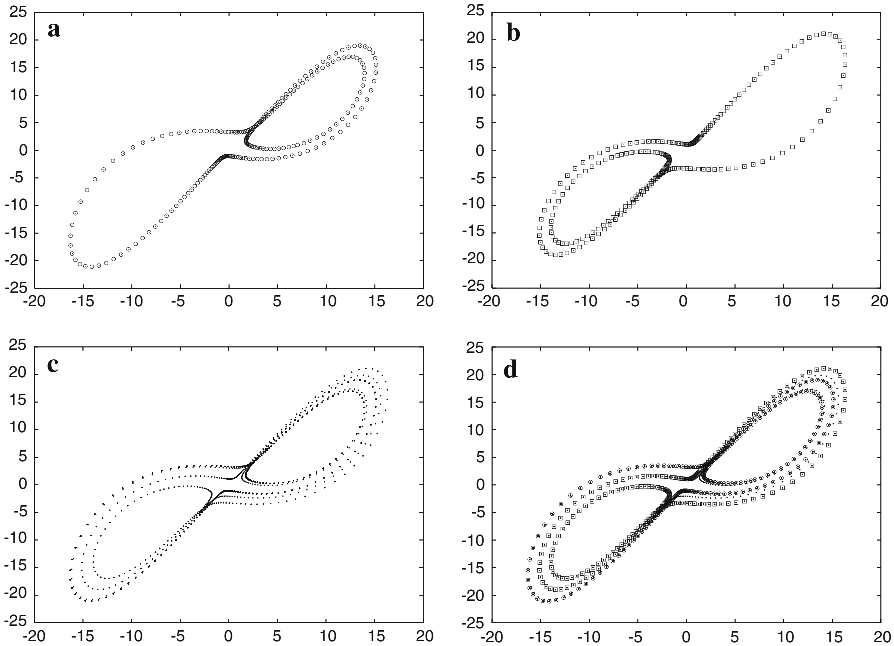

Fig. 3 **a**, **b** Two pseudo periodic orbits of the Lorenz equations for the classic parameter values. **c** A pseudo heteroclinic orbit connecting these two pseudo periodic orbits. **d** The pseudo periodic orbits and the pseudo heteroclinic orbit juxtaposed. There exist true hyperbolic periodic orbits and a true transversal heteroclinic orbit within $\varepsilon_{\mathbf{w}} \leq 7.249 \times 10^{-10}$ of the pseudo ones

7 Infinite shadowing and hyperbolicity

In preparation for the proof of our main theorems, here we state and prove a rather general lemma about shadowing a bounded infinite pseudo orbit by a true hyperbolic orbit. This lemma is significant because the usual uniform hyperbolicity assumption is replaced by the invertibility of the linear operator and several computable inequalities in terms of the constants defined in Sect. 4.

Infinite Shadowing and Hyperbolicity Lemma 7.1. *Let* $\{\mathbf{w}_k\}_{k=-\infty}^{+\infty}$ *be a bounded* δ *pseudo orbit of Eq.* (4.1) *with associated times* $\{h_k\}_{k=-\infty}^{+\infty}$ *such that* $L_{\mathbf{w}}$ *is invertible with* $\|L_{\mathbf{w}}^{-1}\| \leq K$. *Then if*

$$4C\delta < \varepsilon_0, \quad 2M_1C\delta \leq 1, \quad C^2(N_1\delta + N_2\delta^2 + N_3\delta^3) < 1, \tag{7.1}$$

there is a unique true orbit $\{\mathbf{x}_k\}_{k=-\infty}^{+\infty}$ *of Eq.* (4.1) *with associated times* $\{t_k\}_{k=-\infty}^{+\infty}$ *such that for* $k \in Z$

$$f(\mathbf{w}_k)^*(\mathbf{x}_k - \mathbf{w}_k) = 0$$