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Radioactive Decay:
x(t) – amount of material at time t
dx/dt(t) = k x(t)
rate of decay is proportional to the amount present (k depends on material)

Our job is to find a "formula" for x(t) in terms of t.
Experiment:
100 mg Th-234 decays to 82.04 mg in one week. How long will it take to decay to 50 mg?

PHASER -> Equation -> ODE Library -> Cubic 1-D ODE
    Numerics -> Parameters -> hit Cubic 1-D ODE bar to see equation
    $x1=a+bx1+cx1^2+dx1^3$
(a,c, and d are set equal to zero so x1=k*x1 in PHASER)
(k is b in PHASER) -> set b = -0.02828
    Numerics -> Initial Conditions -> x1 = 100
    Numerics -> Time from 1 to 7

If the b value is right, then x1(0)=100 and x1(7)=82.04

xi values

IC1::time	IC1::x1
6.75000E+000	8.2622346773E+001
6.76000E+000	8.2598984477E+001
6.77000E+000	8.2575628787E+001
6.78000E+000	8.2552279701E+001
6.79000E+000	8.2528937217E+001
6.80000E+000	8.2505601334E+001
6.81000E+000	8.2482272048E+001
6.82000E+000	8.2458949360E+001
6.83000E+000	8.2435633266E+001
6.84000E+000	8.2412323765E+001
6.85000E+000	8.2389020855E+001
6.86000E+000	8.2365724534E+001
6.87000E+000	8.2342434801E+001
6.88000E+000	8.2319151653E+001
6.89000E+000	8.2295875088E+001
6.90000E+000	8.2272605105E+001
6.91000E+000	8.2249341702E+001
6.92000E+000	8.2226084877E+001
6.93000E+000	8.2202834628E+001
6.94000E+000	8.2179590953E+001
6.95000E+000	8.2156353850E+001
6.96000E+000	8.2133123318E+001
6.97000E+000	8.2109899355E+001
6.98000E+000	8.2086681959E+001
6.99000E+000	8.2063471127E+001
7.00000E+000	8.2040266859E+001

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Try more time to get 50mg.

b=-0.02828   x1=100   time: 0 – 10

IC1::time	IC1::x1
9.75000E+000	7.5901783791E+001
9.76000E+000	7.5880321802E+001
9.77000E+000	7.5858865881E+001
9.78000E+000	7.5837416027E+001
9.79000E+000	7.5815972238E+001
9.80000E+000	7.5794534512E+001
9.81000E+000	7.5773102848E+001
9.82000E+000	7.5751677245E+001
9.83000E+000	7.5730257699E+001
9.84000E+000	7.5708844210E+001
9.85000E+000	7.5687436776E+001
9.86000E+000	7.5666035396E+001
9.87000E+000	7.5644640066E+001
9.88000E+000	7.5623250787E+001
9.89000E+000	7.5601867555E+001
9.90000E+000	7.5580490370E+001
9.91000E+000	7.5559119229E+001
9.92000E+000	7.5537754131E+001
9.93000E+000	7.5516395075E+001
9.94000E+000	7.5495042058E+001
9.95000E+000	7.5473695078E+001
9.96000E+000	7.5452354135E+001
9.97000E+000	7.5431019226E+001
9.98000E+000	7.5409690350E+001
9.99000E+000	7.5388367505E+001
1.00000E+001	7.5367050689E+001

Not enough time….

b=-0.02828   x1=100   time: 0 – 20

IC1::time	IC1::x1
1.97500E+001	5.7204935864E+001
1.97600E+001	5.7188760595E+001
1.97700E+001	5.7172589900E+001
1.97800E+001	5.7156423778E+001
1.97900E+001	5.7140262227E+001
1.98000E+001	5.7124105245E+001
1.98100E+001	5.7107952832E+001
1.98200E+001	5.7091804987E+001
1.98300E+001	5.7075661707E+001
1.98400E+001	5.7059522992E+001
1.98500E+001	5.7043388840E+001
1.98600E+001	5.7027259251E+001
1.98700E+001	5.7011134222E+001
1.98800E+001	5.6995013753E+001
1.98900E+001	5.6978897842E+001
1.99000E+001	5.6962786488E+001
1.99100E+001	5.6946679689E+001
1.99200E+001	5.6930577445E+001
1.99300E+001	5.6914479754E+001
1.99400E+001	5.6898386615E+001
1.99500E+001	5.6882298026E+001
1.99600E+001	5.6866213987E+001
1.99700E+001	5.6850134495E+001
1.99800E+001	5.6834059550E+001
1.99900E+001	5.6817989151E+001
2.00000E+001	5.6801923295E+001

Still not enough time….

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So far, our time parameters have not shown us when x approaches 50 so we must continue increasing our final time.

b=-0.02828   x1=100   time: 0 – 25

IC1::time	IC1::x1
2.47500E+001	4.9662006598E+001
2.47600E+001	4.9647964168E+001
2.47700E+001	4.9633925709E+001
2.47800E+001	4.9619891219E+001
2.47900E+001	4.9605860698E+001
2.48000E+001	4.9591834144E+001
2.48100E+001	4.9577811556E+001
2.48200E+001	4.9563792934E+001
2.48300E+001	4.9549778275E+001
2.48400E+001	4.9535767579E+001
2.48500E+001	4.9521760844E+001
2.48600E+001	4.9507758070E+001
2.48700E+001	4.9493759256E+001
2.48800E+001	4.9479764400E+001
2.48900E+001	4.9465773501E+001
2.49000E+001	4.9451786558E+001
2.49100E+001	4.9437803570E+001
2.49200E+001	4.9423824536E+001
2.49300E+001	4.9409849454E+001
2.49400E+001	4.9395878325E+001
2.49500E+001	4.9381911145E+001
2.49600E+001	4.9367947915E+001
2.49700E+001	4.9353988634E+001
2.49800E+001	4.9340033299E+001
2.49900E+001	4.9326081910E+001
2.50000E+001	4.9312134467E+001

We went too far - x value seems to be approaching 50 around the 24th iteration.

# of Last Xi values to View increased to 250

IC1::time	IC1::x1
2.43900E+001	5.0170188012E+001
2.44000E+001	5.0156001889E+001
2.44100E+001	5.0141819777E+001
2.44200E+001	5.0127641675E+001
2.44300E+001	5.0113467582E+001
2.44400E+001	5.0099297498E+001
2.44500E+001	5.0085131419E+001
2.44600E+001	5.0070969347E+001
2.44700E+001	5.0056811279E+001
2.44800E+001	5.0042657214E+001
2.44900E+001	5.0028507151E+001
2.45000E+001	5.0014361090E+001
2.45100E+001	5.0000219028E+001
2.45200E+001	4.9986080966E+001
2.45300E+001	4.9971946901E+001
2.45400E+001	4.9957816832E+001
2.45500E+001	4.9943690759E+001
2.45600E+001	4.9929568680E+001
2.45700E+001	4.9915450595E+001
2.45800E+001	4.9901336501E+001
2.45900E+001	4.9887226398E+001
2.46000E+001	4.9873120285E+001
2.46100E+001	4.9859018161E+001
2.46200E+001	4.9844920024E+001
2.46300E+001	4.9830825874E+001
2.46400E+001	4.9816735709E+001
2.46500E+001	4.9802649528E+001
2.46600E+001	4.9788567330E+001
2.46700E+001	4.9774489114E+001
2.46800E+001	4.9760414879E+001
2.46900E+001	4.9746344623E+001
2.47000E+001	4.9732278346E+001
2.47100E+001	4.9718216046E+001
2.47200E+001	4.9704157722E+001
2.47300E+001	4.9690103374E+001
2.47400E+001	4.9676052999E+001

It takes around 24.5 days to decay to ½ (50mg). So the **half-life** of Th-234 is about 24.5 days.

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Two half-lives would be about 49 days. So in 49 days, we would have ¼ of our starting value. If time is increased (to be about 49.2), we can see this.

b=-0.02828   x1=250   time: 0 – 49.2   (250 x ¼ = 62.5)

IC1::time	IC1::x1
4.88600E+001	6.2783990828E+001
4.88700E+001	6.2766238026E+001
4.88800E+001	6.2748490244E+001
4.88900E+001	6.2730747480E+001
4.89000E+001	6.2713009733E+001
4.89100E+001	6.2695277001E+001
4.89200E+001	6.2677549283E+001
4.89300E+001	6.2659826579E+001
4.89400E+001	6.2642108885E+001
4.89500E+001	6.2624396201E+001
4.89600E+001	6.2606688526E+001
4.89700E+001	6.2588985858E+001
4.89800E+001	6.2571288195E+001
4.89900E+001	6.2553595537E+001
4.90000E+001	6.2535907881E+001
4.90100E+001	6.2518225227E+001
4.90200E+001	6.2500547572E+001
4.90300E+001	6.2482874917E+001
4.90400E+001	6.2465207258E+001
4.90500E+001	6.2447544595E+001
4.90600E+001	6.2429886926E+001
4.90700E+001	6.2412234250E+001
4.90800E+001	6.2394586566E+001
4.90900E+001	6.2376943872E+001
4.91000E+001	6.2359306166E+001
4.91100E+001	6.2341673448E+001
4.91200E+001	6.2324045715E+001
4.91300E+001	6.2306422967E+001
4.91400E+001	6.2288805202E+001
4.91500E+001	6.2271192418E+001
4.91600E+001	6.2253584615E+001
4.91700E+001	6.2235981790E+001
4.91800E+001	6.2218383943E+001
4.91900E+001	6.2200791072E+001
4.92000E+001	6.2183203175E+001

At slightly more than 49 days, our x value is almost exactly 62.5.

<u>Radio Carbon dating</u>

C12 is a stable isotope of carbon, but C14 is unstable so it undergoes radioactive decay. The half-life of carbon-14 is about 5700 years. Carbon-14 levels in old objects combined with the known half-life (rate of decay) are used to determine age.

Shroud of Turin: historical relic that supposedly covered Christ before he rose.
Lab experiments show that it contained 92% of the carbon-14 in living matter, meaning 8% had decayed.

Using PHASER and the Cubic 1-D ODE equation:

Numerics -> Parameters -> set b= -0.0001216
Numerics -> Initial Conditions -> x1=100 (since we know there should be 92% left)

Experimented with times of 0 to 2000, 0 to 1000, 0 to 700, 0 to 695, and 0 to 693. The Xi values (use the Xi Values view) show xi values that display too much decay to match the 92% of the Shroud of Turin

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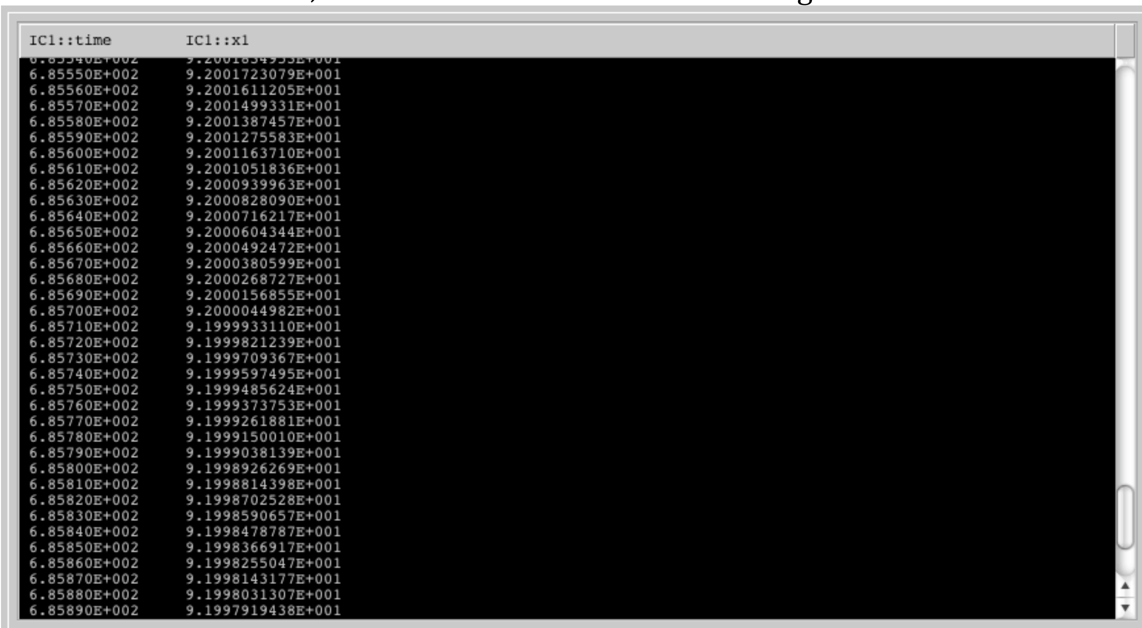
Adjusting time shows that there is 92% of carbon-14 remaining after 685.7 years. Since this experiment was done in 1988: 1988-685.7 = 1303.
So this Shroud of Turin, seems to date back to the Middle Ages.



IC1::time	IC1::x1
6.85550E+002	9.2001723079E+001
6.85560E+002	9.2001611205E+001
6.85570E+002	9.2001499331E+001
6.85580E+002	9.2001387457E+001
6.85590E+002	9.2001275583E+001
6.85600E+002	9.2001163710E+001
6.85610E+002	9.2001051836E+001
6.85620E+002	9.2000939963E+001
6.85630E+002	9.2000828090E+001
6.85640E+002	9.2000716217E+001
6.85650E+002	9.2000604344E+001
6.85660E+002	9.2000492472E+001
6.85670E+002	9.2000380599E+001
6.85680E+002	9.2000268727E+001
6.85690E+002	9.2000156855E+001
6.85700E+002	9.2000044982E+001
6.85710E+002	9.1999933110E+001
6.85720E+002	9.1999821239E+001
6.85730E+002	9.1999709367E+001
6.85740E+002	9.1999597495E+001
6.85750E+002	9.1999485624E+001
6.85760E+002	9.1999373753E+001
6.85770E+002	9.1999261881E+001
6.85780E+002	9.1999150010E+001
6.85790E+002	9.1999038139E+001
6.85800E+002	9.1998926269E+001
6.85810E+002	9.1998814398E+001
6.85820E+002	9.1998702528E+001
6.85830E+002	9.1998590657E+001
6.85840E+002	9.1998478787E+001
6.85850E+002	9.1998366917E+001
6.85860E+002	9.1998255047E+001
6.85870E+002	9.1998143177E+001
6.85880E+002	9.1998031307E+001
6.85890E+002	9.1997919438E+001

How does PHASER generate these numbers? How good are these numbers?
Example: x1'=x1 (k=1) ← a number whose derivative is itself

guess: $x1(t)=e^t$

satisfies: x1(0)=1   initial condition

interesting: x1(1)=e

What is e???

To analyze using PHASER, use Cubic 1-D ODE equation with:
Numerics -> Parameters -> b=1.0
Numerics -> Initial Conditions -> x1=1.0
Numerics -> Time -> 0 to 1
Numerics -> Current View -> Xi Mantissa Fraction Precision -> 15

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IC1::time	IC1::x1
[illegible]	[illegible]
6.70000E-001	1.954237320635977E+000
6.80000E-001	1.973877732230486E+000
6.90000E-001	1.993715533243121E+000
7.00000E-001	2.013752707470517E+000
7.10000E-001	2.033991258646792E+000
7.20000E-001	2.054433210643930E+000
7.30000E-001	2.075080607674166E+000
7.40000E-001	2.095935514494409E+000
7.50000E-001	2.117000016612720E+000
7.60000E-001	2.138276220496865E+000
7.70000E-001	2.159766253784963E+000
7.80000E-001	2.181472265498250E+000
7.90000E-001	2.203396426255987E+000
8.00000E-001	2.225540928492518E+000
8.10000E-001	2.247907986676523E+000
8.20000E-001	2.270499837532459E+000
8.30000E-001	2.293318740264237E+000
8.40000E-001	2.316366976781147E+000
8.50000E-001	2.339646851926048E+000
8.60000E-001	2.363160693705853E+000
8.70000E-001	2.386910853524336E+000
8.80000E-001	2.410899706417271E+000
8.90000E-001	2.435129651289937E+000
9.00000E-001	2.459603111157013E+000
9.10000E-001	2.484322533384882E+000
9.20000E-001	2.509290389936364E+000
9.30000E-001	2.534509177617922E+000
9.40000E-001	2.559981418329341E+000
9.50000E-001	2.585709659315917E+000
9.60000E-001	2.611696473423189E+000
9.70000E-001	2.637944459354225E+000
9.80000E-001	2.664456241929492E+000
9.90000E-001	2.691234472349338E+000
1.00000E+000	2.718281828459122E+000

The Xi values go to 2.718281828459122. So,

e= 2.718281828459122

<u>Algorithms for solving ODEs</u>

ODE = ordinary differential equation

given $^{dx}/_{dt}(t)=f(x(t))$ and $x(o)=x0$

Math tells us (if f is a nice function) that there is a unique function x(t) that satisfies these two conditions.

Consider: a coordinate graph with time (t) as the x axis and our x value (x(t)) as the y axis. (x(t0)=x0)

To make this function discrete, we must chop it into steps (different times – t1, t2, t3, etc.) with particular x values at each time.

To get a representation of x(t) in the computer, compute x(t) at discrete time intervals.

Graph many points – looks continuous; otherwise interpolate

Example: $^{dx}/_{dt}=x$, $x(0)=1$

We need to choose how finely we want to chop up our time intervals (choose a <u>step size</u>).

Step size = distance between consecutive time intervals.

If step size is too small, answers are not accurate. Using only 10 steps in PHASER to once again approximate e, we got an answer of 2.593742460100000. Increasing number of steps increases accuracy.