

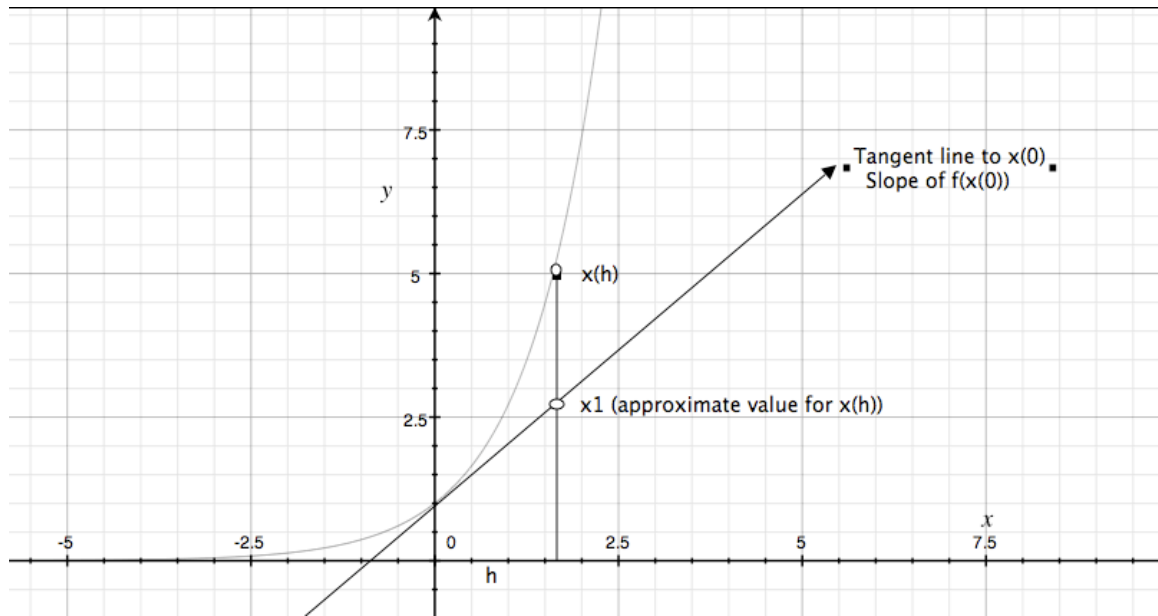
10/1/09

### Euler's Equation

Given  $dx/dt=f(x)$  Initial condition  $x(0)=x_0$

Choose a time step  $h$

We want to find an approximate value for  $x(h)$



Take

$x_1$  (intersection of tangent line and vertical at  $t=h$ ) to be the approximate value for  $x(h)$ .

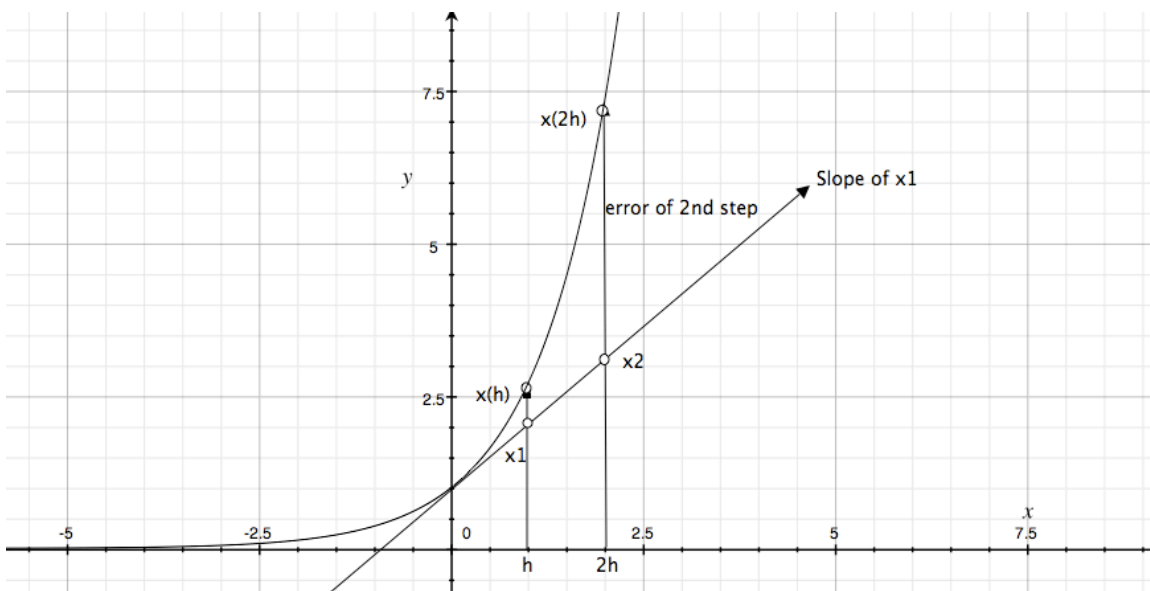
We want to find a formula for  $x$ , in terms of known quantities

$$\text{Slope of tangent} = \text{rise/run} = f(x_0) = (x_1 - x_0) / h$$

$$x_1 - x_0 = h f(x_0)$$

$$x_1 = x_0 + h f(x_0) \text{ (the right hand side is computable! Only approximate value)}$$

### How to take second step



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In general  $x_{n+1} = x_n + hf(x_n)$

Take  $x_n \approx x(nh)$  as approximation

We will address the validity of accuracy of this approximation.

Example:  $dx/dt = x = f(x)$  Compute  $x(1) = ?$  When  $x(0) = 1$

Let us take  $h=1$  So we get there in one step ( $t=1$ )

$$x_1 = x_0 + hf(x_0)$$

$$x_1 = 1 + 1(1) = 2 \quad \text{The exact solution is } x(t) = e^t \quad \text{So } x(1) = e = 2.718281828\dots$$

Therefore the error is  $\text{abs}(2-e)$

Take  $h=0.5$  (must take 2 steps to get to  $x(1)$ )

$$x_1 = x_0 + hf(x_0)$$

$$x_1 = 1 + 0.5(1) = 1.5$$

Next step =  $x_2 = x_1 + hf(x_1)$

$$x_2 = 1.5 + 0.5(1.5) = 2.25 \quad \text{This is the new approximation for } e$$

New Error  $\text{abs}(2.25-e)$

**As  $h$  gets smaller, error “appears” to get smaller. BUT with smaller  $h$  one must take more steps so computation work increases.**

Cubic 1-D ODE

Numerics -> Parameters ( $b=1$ ), Time (0-1) Initial Conditions ( $x=1$ )

Algorithms -> Classic Explicit -> Euler (1), Step size = 1.0

Xi Values Screen

IC1::time	IC1::x1
0.00000E+000	1.0000000000E+000
1.00000E+000	2.0000000000E+000

Change Step size to 0.5,

IC1::time	IC1::x1
0.00000E+000	1.0000000000E+000
5.00000E-001	1.5000000000E+000
1.00000E+000	2.2500000000E+000

then to 0.1,

IC1::time	IC1::x1
0.00000E+000	1.0000000000E+000
1.00000E-001	1.1000000000E+000
2.00000E-001	1.2100000000E+000
3.00000E-001	1.3310000000E+000
4.00000E-001	1.4641000000E+000
5.00000E-001	1.6105100000E+000
6.00000E-001	1.7715610000E+000
7.00000E-001	1.9487171000E+000
8.00000E-001	2.1435888100E+000
9.00000E-001	2.3579476910E+000
1.00000E+000	2.5937424601E+000

then to 0.001 (to get more digits closer to e)

IC1::time	IC1::x1
9.75000E-001	2.6498759425E+000
9.76000E-001	2.6525258185E+000
9.77000E-001	2.6551783443E+000
9.78000E-001	2.6578335226E+000
9.79000E-001	2.6604913561E+000
9.80000E-001	2.6631518475E+000
9.81000E-001	2.6658149993E+000
9.82000E-001	2.6684808143E+000
9.83000E-001	2.6711492952E+000
9.84000E-001	2.6738204445E+000
9.85000E-001	2.6764942649E+000
9.86000E-001	2.6791707592E+000
9.87000E-001	2.6818499299E+000
9.88000E-001	2.6845317799E+000
9.89000E-001	2.6872163116E+000
9.90000E-001	2.6899035279E+000
9.91000E-001	2.6925934315E+000
9.92000E-001	2.6952860249E+000
9.93000E-001	2.6979813109E+000
9.94000E-001	2.7006792922E+000
9.95000E-001	2.7033799715E+000
9.96000E-001	2.7060833515E+000
9.97000E-001	2.7087894349E+000
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9.99000E-001	2.7142097225E+000
1.00000E+000	2.7169239322E+000

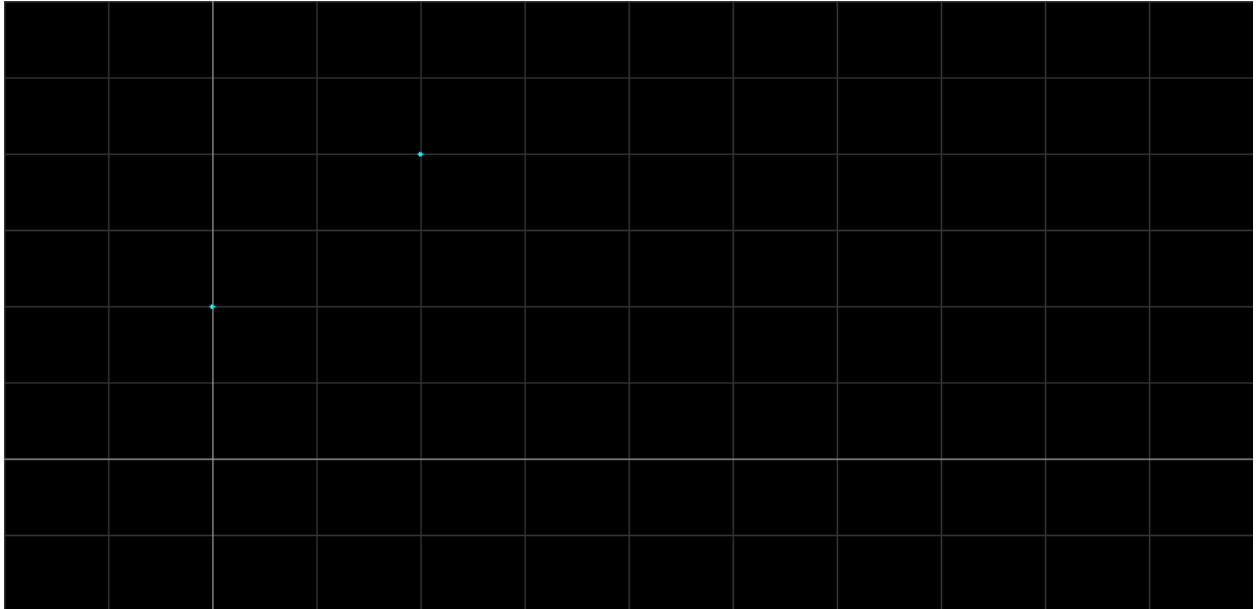
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$\Xi$  vs. Time Screen

Plot exponential function, Set window size (-1,5) (-1,3)

Axis Grids (x=0.5, y=0.5) Want to make square grids

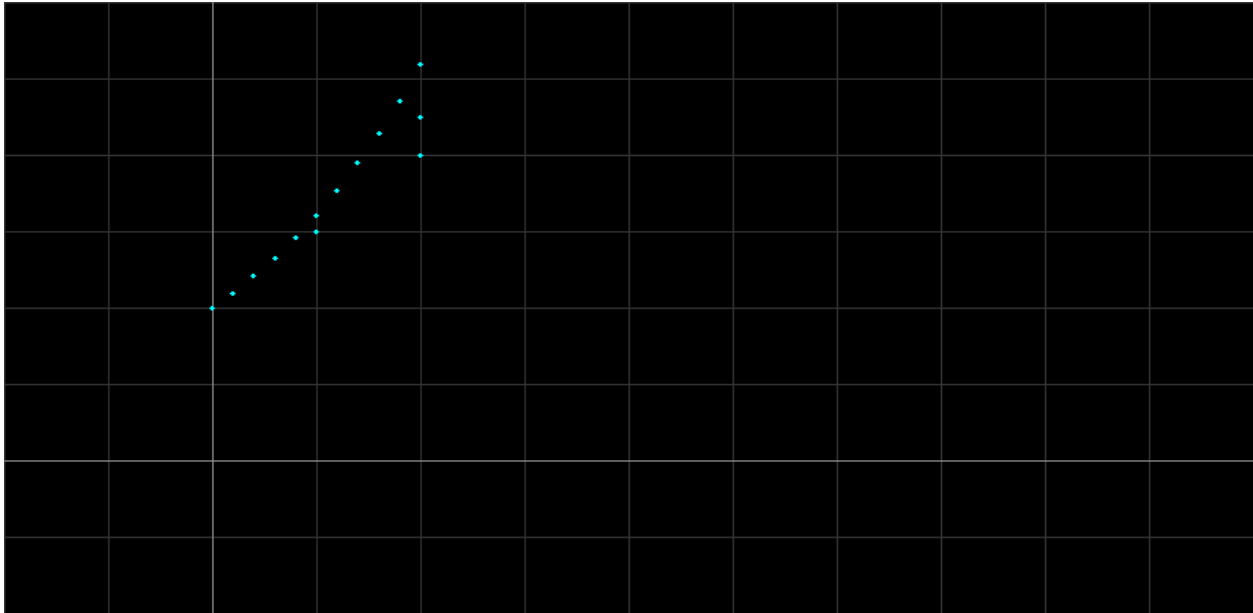
When Step size 1.0



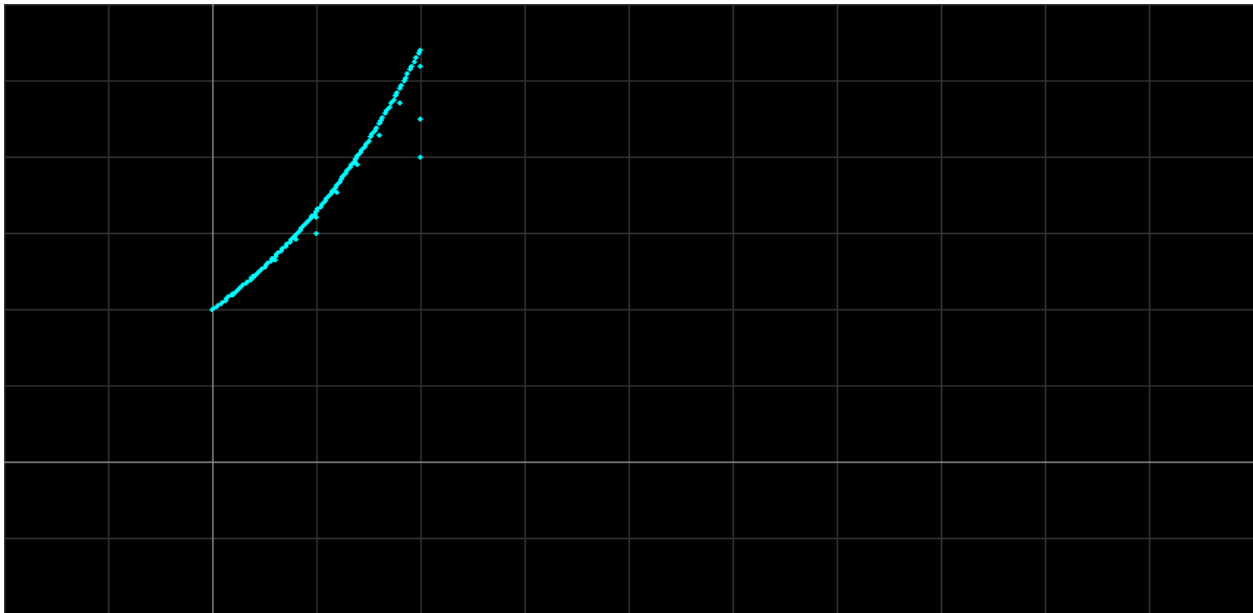
When Step size 0.5 (overlapped with graph of step size 1.0)



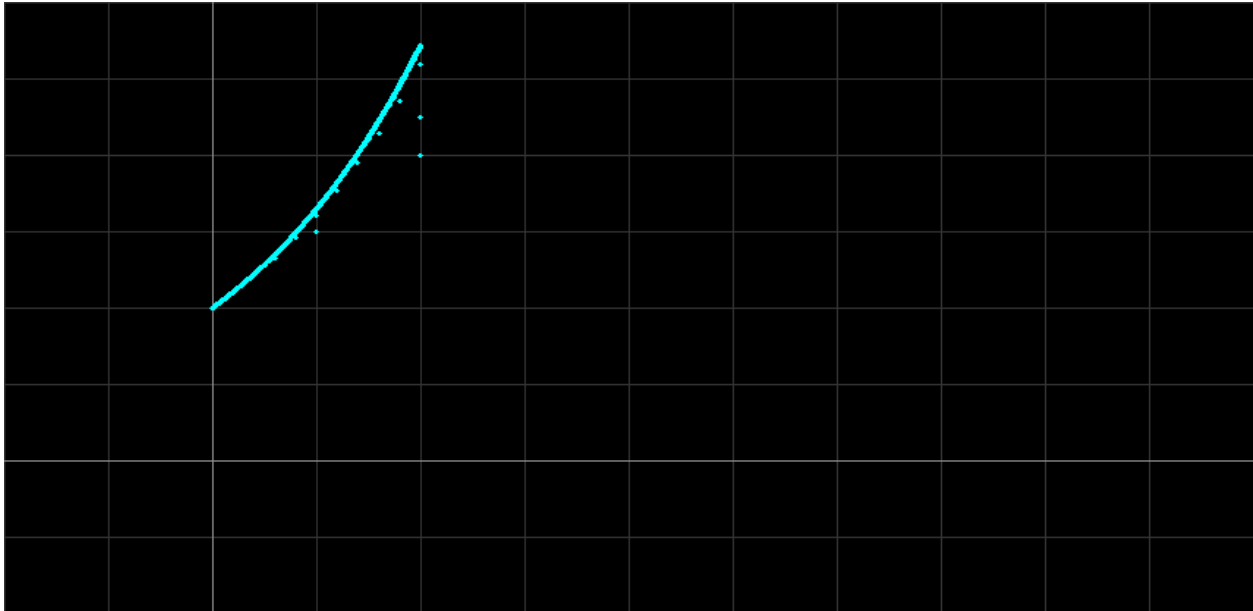
When step size 0.1 (overlapped with first 2 graphs)



When step size 0.01 (overlapped with first 3 graphs)]



When step size 0.001 (overlapped with first 4 graphs)



Once we have  $dx/dt=x$   $x(0)=x_0=0.5$  (initial condition changed!)

What is  $x(t)$ ?

$$dx/dt=x_0e^t=x$$

$$x(0)=x_0e^0=x_0$$

When initial condition is 0.5 function is  $0.5e^t=f(t)$

### Direction Fields

Xi vs Time -> Numerics -> Current View-> Direction Field

