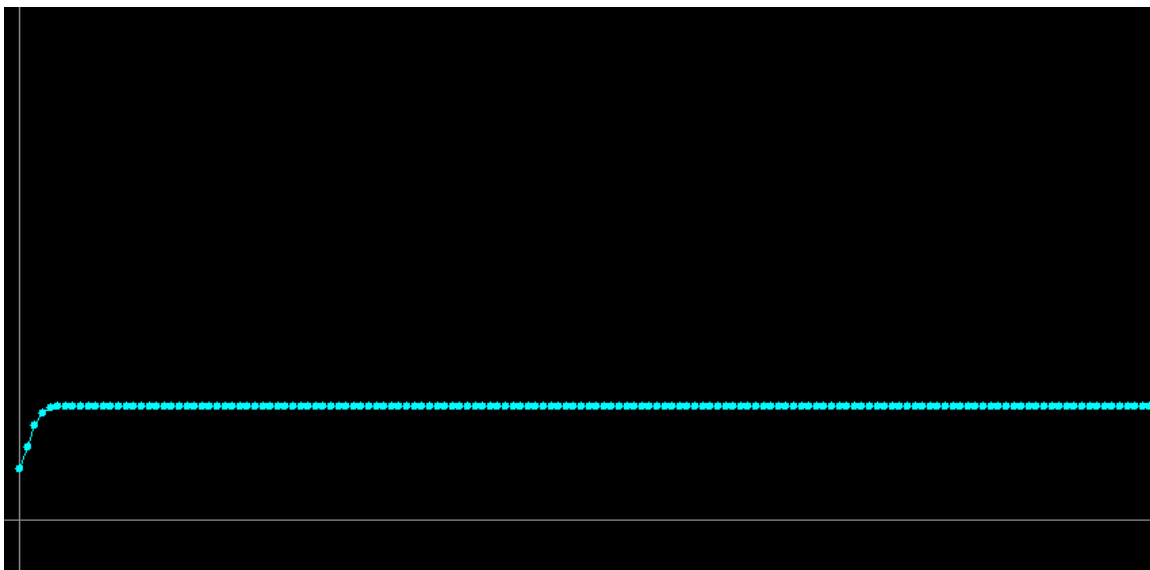


CSC 210 9/22/09

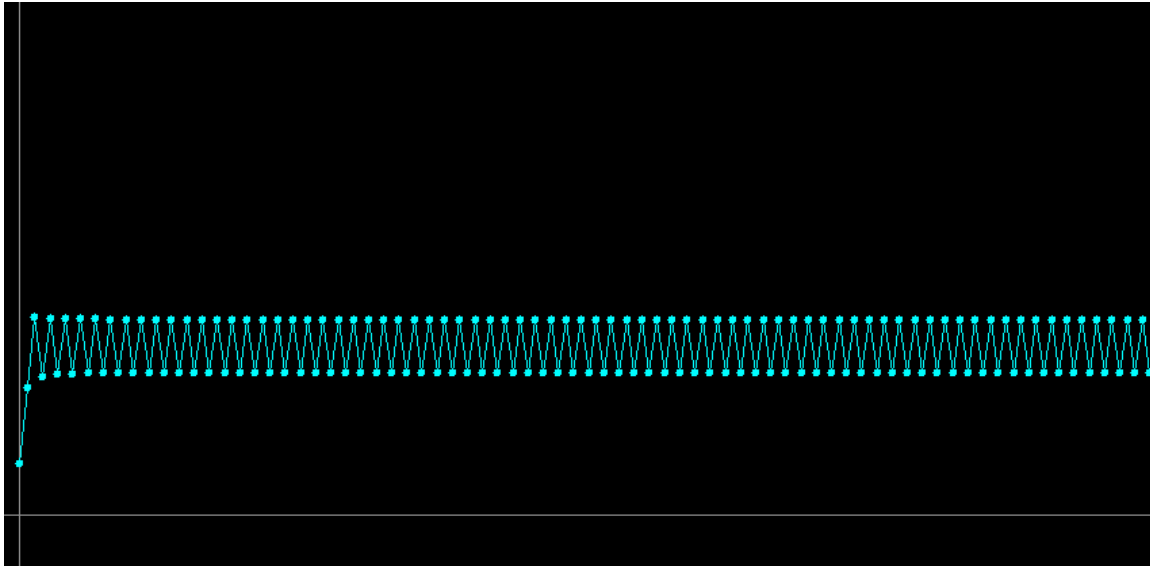
## Logistic MAP Equation

The logistic MAP Equation is a model for population density.

- Equation -> MAP Library -> Logistic MAP
- View -> Xi vs. Time
- Equation -> Load Equation Defaults
- Numerics -> Parameters -> set  $a = 1.8$ ;
  - Time from 0 to 150
  - Current View -> Graphics -> Window Size -> x-axis -2.0 to 150, y-axis -0.2 to 2
  - (If desired) Graphics -> change graph point size and connect points

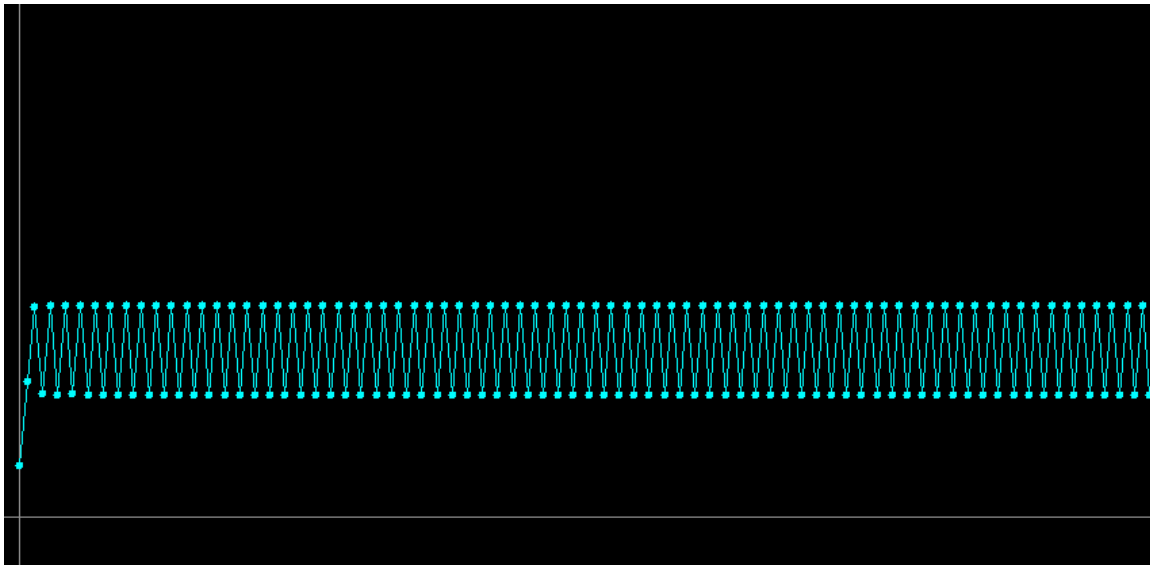


Set  $a = 3.1$

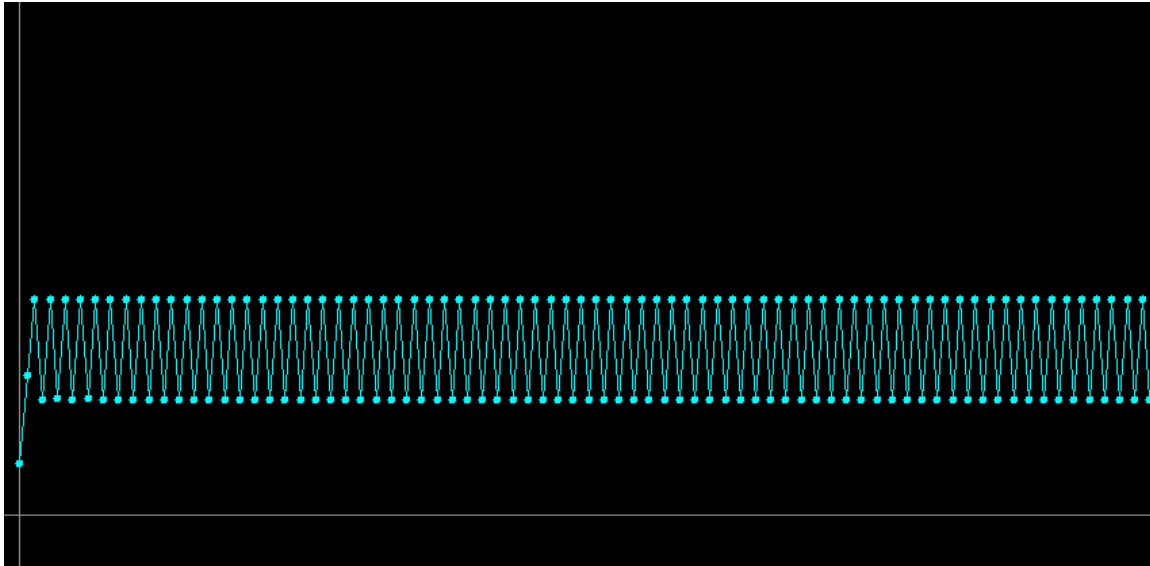


Two sets of points?  $\rightarrow$  oscillation (use graphics to connect the points to make sure they are not overlapping each other)

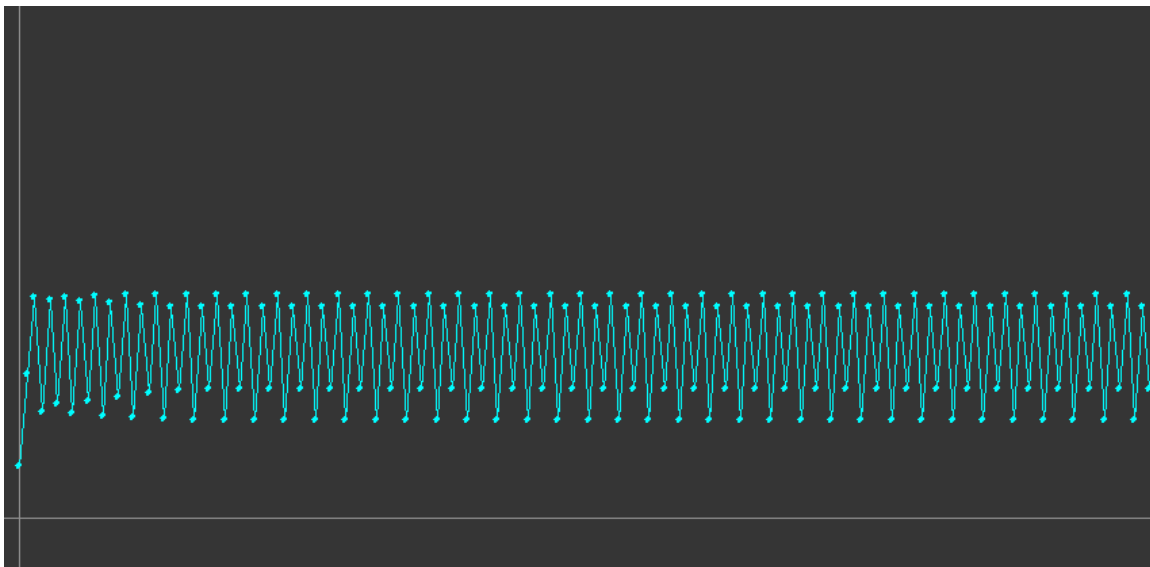
Set  $a=3.3$



Set  $a = 3.4$

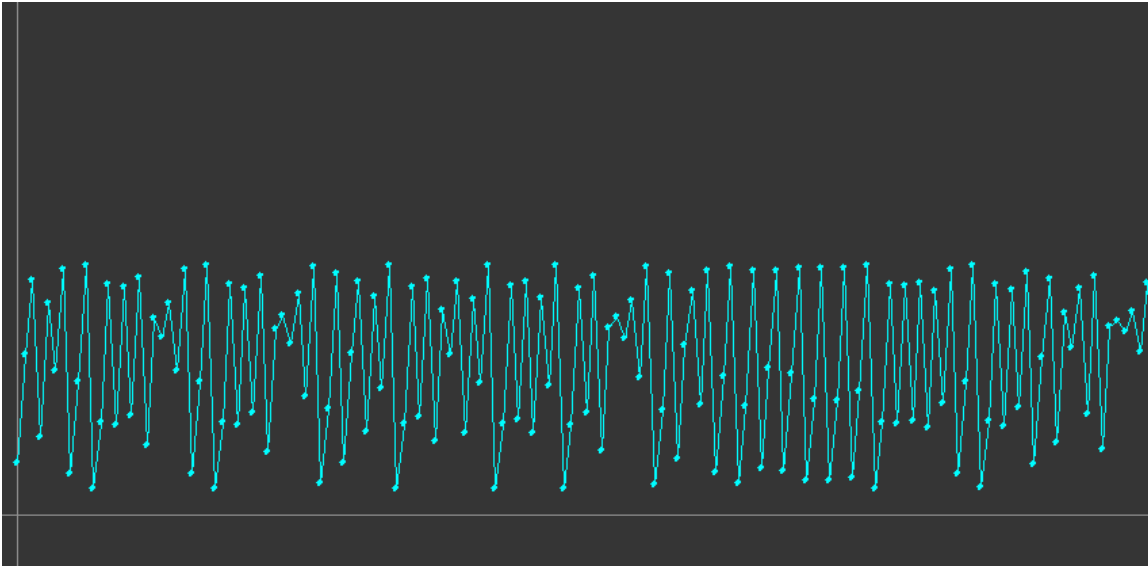


Set  $a = 3.5$



This is a period-4 oscillation. Every fourth generation is the same.

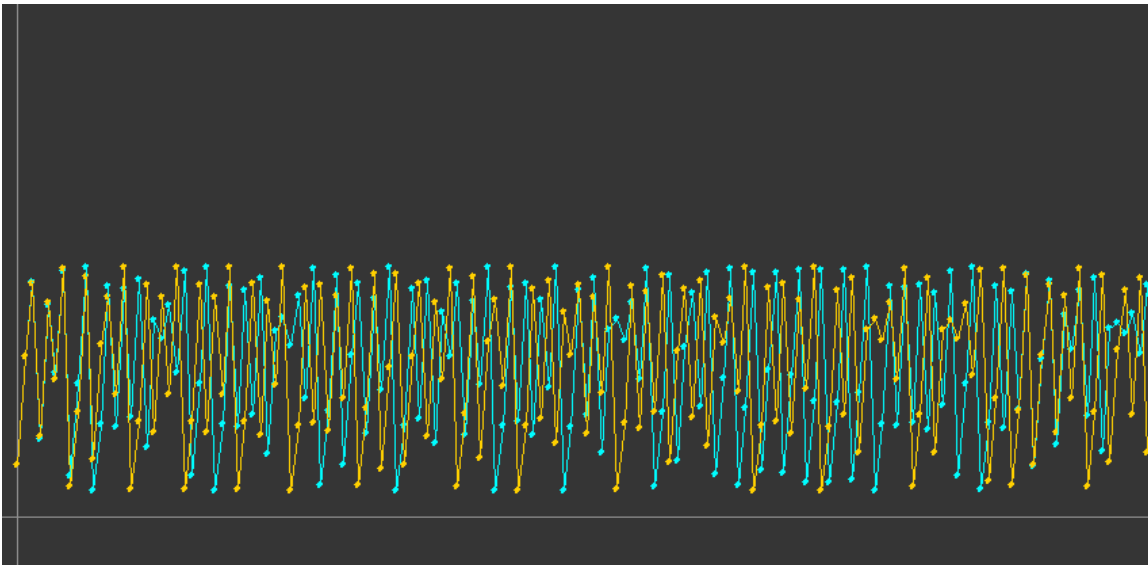
Set  $a = 3.895$



This is a chaotic time series. It looks like it may be oscillating, but really it is not following a pattern.

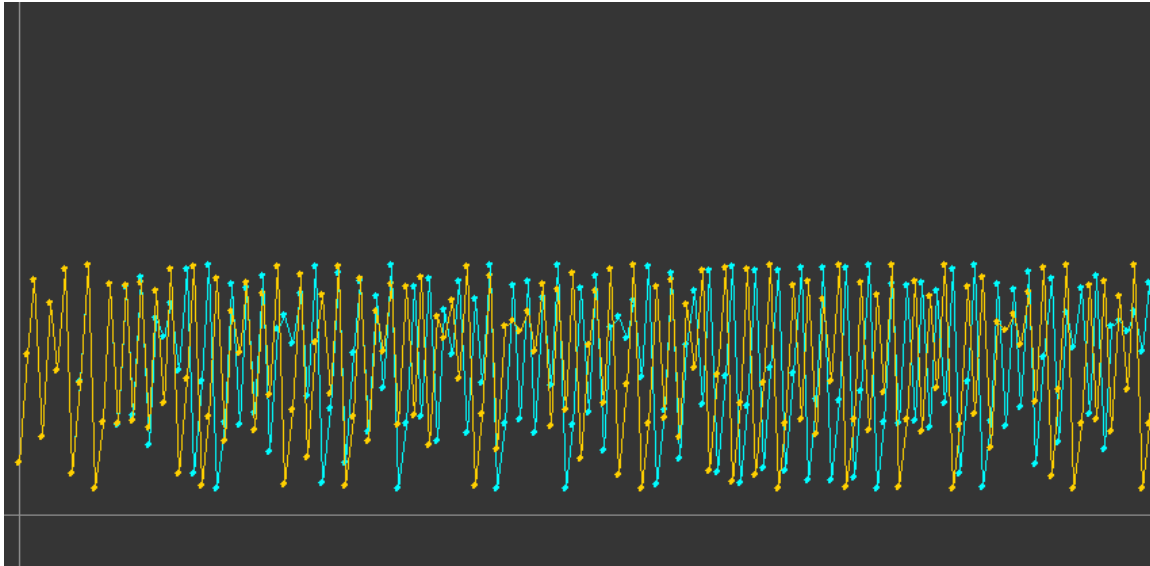
Numerics -> Initial Conditions -> click box for set #2

Set  $t=0$  and  $x_1=0.201$



Even though the initial conditions only had a difference of 0.001, it continued to make a difference throughout later generations.

Change  $x_1=0.20001$



These two initial conditions have an even smaller difference (.00001) but they are even more out of sync than the initial conditions 0.2 and 0.201

This means that there is *sensitive dependence on initial data*.

Bifurcation diagram: plots a parameter (like  $a$  or  $r$ ) on the x-axis and the  $x_i$  value on the y-axis.

- Fix an initial condition
- Fix a parameter value
- Iterate and plot
- Increase parameter a little and do it again

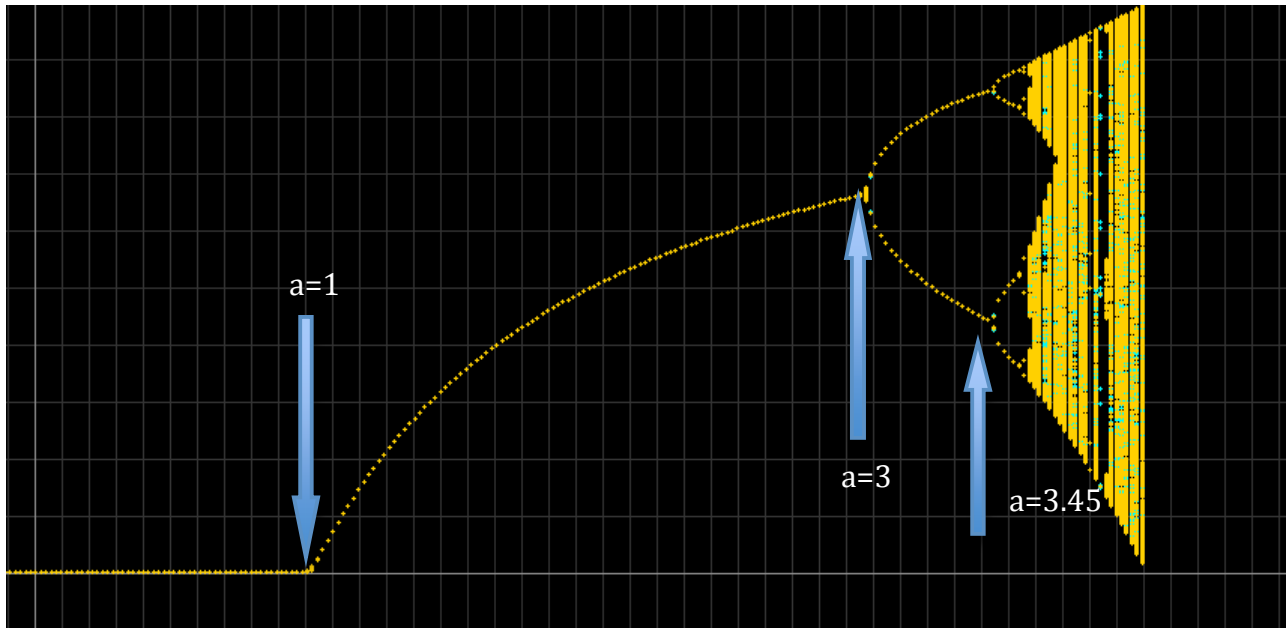
View -> Bifurcation Diagram

$$x_{n+1} = a * x_n (1 - x_n)$$

where  $0 < x_n < 1$  (the y - axis)

and  $a$  is a positive parameter (the x - axis)

Time -> leave out the transients, set time from 100 to 1000



This bifurcation diagram shows that for the parameter  $0 \leq a < 1$  the population goes to zero. (Biologically, the population is decreasing, and eventually there will be none left.)

For the parameter  $1 \leq a \leq 3$ , the population settles to an asymptotically stable fixed point.

As the parameter gets larger, so does the value of the fixed point.

For the parameter  $3 \leq a < 4$  the population oscillates. The larger the  $a$ , the bigger the oscillation.

At  $a = 3$ , there is the first period doubling (period - 2)

At  $a \approx 3.45$ , there is a second period doubling (period - 4)

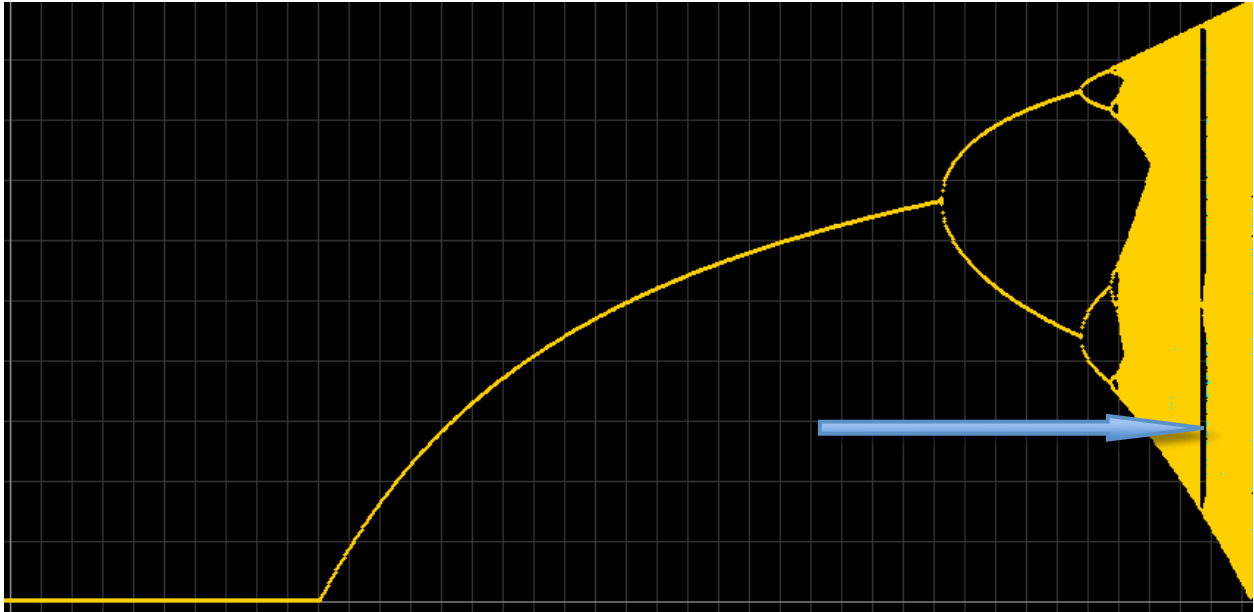
Continues to split until it becomes chaotic

$a > 4$  is unrealistic

Set Window Size: x-axis -> -0.2 to 4

Current View -> Bifurcation -> Parameter Sample Size -> 1000

Time -> 1000 to 2500



Notice that there are small regions where the diagram leaves chaos and enters into an orderly region before going back into chaos

(In Phaser main view on bottom toolbar, you can set the mouse from IC (initial conditions) to ZOOM and click and drag a region to zoom in and out on your graph.)