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Abstract algebra is not only a major subject of science, but it is also magic and fun. Abstract algebra is not all work and no play, and it is certainly not a dull boy. See, for example, the neat card trick on page 18. This trick is based, not on sleight of hand, but rather on a theorem in abstract algebra. Anyone can do it, but to understand it you need some group theory. And before beginning the course, you might first try your skills on the famous (some would say infamous) tile puzzle. In this puzzle, a frame has 12 spaces, the first 11 with numbered tiles and the last vacant. The last two tiles are out of order. Is it possible to slide the tiles around to get them all in order, and end again with the last space vacant? After giving up on this, you can study permutation groups and learn the answer!