

Outline

Chapter 1 Background and Fundamentals of Mathematics

Sets, Cartesian products	1
Relations, partial orderings, Hausdorff maximality principle, equivalence relations	3
Functions, bijections, strips, solutions of equations, right and left inverses, projections	5
Notation for the logic of mathematics	13
Integers, subgroups, unique factorization	14

Chapter 2 Groups

Groups, scalar multiplication for additive groups	19
Subgroups, order, cosets	21
Normal subgroups, quotient groups, the integers mod n	25
Homomorphisms	27
Permutations, the symmetric groups	31
Product of groups	34

Chapter 3 Rings

Rings	37
Units, domains, fields	38
The integers mod n	40
Ideals and quotient rings	41
Homomorphisms	42
Polynomial rings	45
Product of rings	49
The Chinese remainder theorem	50
Characteristic	50
Boolean rings	51

Chapter 4 Matrices and Matrix Rings

Addition and multiplication of matrices, invertible matrices	53
Transpose	56
Triangular, diagonal, and scalar matrices	56
Elementary operations and elementary matrices	57
Systems of equations	59

Determinants, the classical adjoint	60
Similarity, trace, and characteristic polynomial	64
Chapter 5 Linear Algebra	
Modules, submodules	68
Homomorphisms	69
Homomorphisms on R^n	71
Cosets and quotient modules	74
Products and coproducts	75
Summands	77
Independence, generating sets, and free basis	78
Characterization of free modules	79
Uniqueness of dimension	82
Change of basis	83
Vector spaces, square matrices over fields, rank of a matrix	85
Geometric interpretation of determinant	90
Linear functions approximate differentiable functions locally	91
The transpose principle	92
Nilpotent homomorphisms	93
Eigenvalues, characteristic roots	95
Jordan canonical form	96
Inner product spaces, Gram-Schmidt orthonormalization	98
Orthogonal matrices, the orthogonal group	102
Diagonalization of symmetric matrices	103
Chapter 6 Appendix	
The Chinese remainder theorem	108
Prime and maximal ideals and UFD ^s	109
Splitting short exact sequences	114
Euclidean domains	116
Jordan blocks	122
Jordan canonical form	123
Determinants	128
Dual spaces	130

1	2	3	4
5	6	7	8
9	11	10	

Abstract algebra is not only a major subject of science, but it is also magic and fun. Abstract algebra is not all work and no play, and it is certainly not a dull boy. See, for example, the neat card trick on page 18. This trick is based, not on sleight of hand, but rather on a theorem in abstract algebra. Anyone can do it, but to understand it you need some group theory. And before beginning the course, you might first try your skills on the famous (some would say infamous) tile puzzle. In this puzzle, a frame has 12 spaces, the first 11 with numbered tiles and the last vacant. The last two tiles are out of order. Is it possible to slide the tiles around to get them all in order, and end again with the last space vacant? After giving up on this, you can study permutation groups and learn the answer!