

Syllabus for MTH 782: Topics in Topology

University of Miami, Fall 2023

www.math.miami.edu/~cscaduto/teaching/782-fall-2023/

Instructor: Prof. Christopher Scaduto

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Office hours: Tues 8:15-9:15, Thurs 11-12, or by appointment

Class time and location:

Tuesday and Thursday 12:30PM - 1:45PM, Ungar 406

References:

There is no textbook for the course. References for material covered are given on the course webpage at the URL above.

Description:

In this course we study applications of gauge theory to the topology of low-dimensional manifolds, primarily 4-manifolds. We will survey Freedman's classification of simply-connected topological 4-manifolds, and describe some famous results about smooth 4-manifolds that were proved by Donaldson using Yang–Mills instanton gauge theory. We give enough background in gauge theory to prove Donaldson's "Diagonalization Theorem", and then define the Donaldson invariants of smooth 4-manifolds. Following the historical record, we then introduce Seiberg–Witten invariants and prove the Thom Conjecture about surfaces in the complex projective plane, following Kronheimer and Mrowka. In the remaining time, we will cover some basics of Floer 3-theory for 3-manifolds and/or knots.

The amount of mathematics involved in the topics listed above is vast. To cover the above, we will be forced to skip numerous details. Most notably, we will avoid doing any analysis (while still describing enough moduli theory needed for applications). I will give precise references throughout the course indicating where more details can be found.

Grading:

You will choose a topic of your choice related to gauge theory and/or 4-manifold topology and write an expository account of your topic, working out some problems. This is a very large area of study and is connected to most other major fields of mathematics (analysis, differential equations, combinatorics, algebraic geometry, ...). Please come to me for help choosing a topic. For credit, you must: run your topic choice by me before submission, and show me a draft (a few weeks prior to the end of the semester, on a specified date).

Standard university policies are assumed for this course. See for example

<https://bulletin.miami.edu/general-university-information/graduate-policies-and-procedures/academic-policies/>