Writing Project

Due date: Thursday, May 9, at 5 PM.

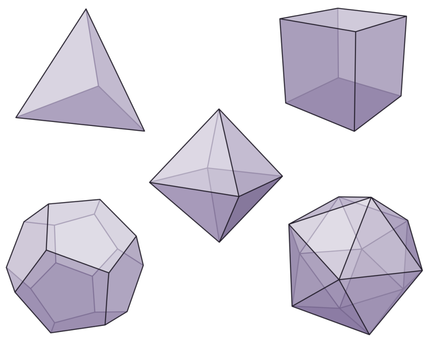
This writing project is optional for the course; you will receive writing credit for the course upon completing this project satisfactorily.

Synopsis: Choose a topic related to the course material and write a paper on it. The topic should illustrate some of the important concepts we have learned this semester. A "paper" means an expository essay with perhaps as much math as a typical homework assignment. In contrast to a homework assignment, this writing should explain the significance of computations in words, and also add commentary to the mathematics. Write about the topic as if you are to be explaining it to yourself in a year or two.

You should talk to me about your project: what it might contain, what references may be useful, etc. Feel free to use office hour time to discuss with me. **A good way to ensure that you receive credit for this project is to submit a draft to me at least 2 weeks before the due date, so that I can provide any necessary feedback.**

Below are listed some example topics. You do not need to choose one of these topics. However, if you choose your own topic, you should run it by me.

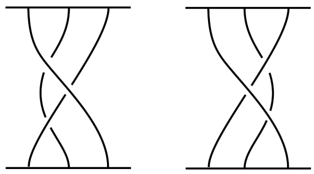
- Explore the rotational symmetries of one or more of the platonic solids that we did not cover in class. There are five such solids:

  We studied the tetrahedron and the cube. In studying the symmetry group of a platonic solid, you should explain the different types of symmetries (and include some pictures), and realize the group as a subgroup of permutations. You can then also study some of its subgroups (which ones are normal?) and look for any homomorphisms to smaller groups. (Compare Note 16.)

- Explore the applications of group theory to cryptography. In particular, you can investigate what we touched upon in Note 11, which described the RSA cryptosystem. You can explain in your own words the algorithm, and also do a number of computations to illustrate how it works, and why it works well.

- Define and explore the *Braid groups*. The elements of the braid group B_n are pictures representing n strings in 3-dimensional space which go from top to bottom.



For example, the above two pictures represent the same element in the Braid group B_3. Define the Braid group and show that with the right operation (stacking braids) it is indeed a group. You can then explore its connection to the symmetric group S_n, among other things.

- An *automorphism* of a group G is a 1-1 and onto homomorphism $\phi : G \to G$. Show that the automorphisms of a group form a group, $\mathrm{Aut}(G)$. Introduce the concept of inner automorphism. Compute $\mathrm{Aut}(G)$ for $G = \mathbb{Z}_n$, $G = Q_8$ and maybe some other interesting groups. You may also consider automorphisms of rings.

- In the spirit of the last suggestion, you can study any topic in group theory that we did not cover in class. Standard topics include Group Actions and the Sylow subgroup theorems. (Both found in most textbooks.)

- The quaternions are a generalization of the complex numbers; to get them we had to give up commutativity. There is a generalization of the quaternions to what are called the *octonions*; however, to get them, you have to give up associativity! Define the octonions and study their basic properties. Discuss the connections to group theory.

- Continue our investigation of algebraic varieties that was begun in Note 26. Prove some of the things we left out, and include a few results we did not mention. Illustrate any results with your some examples.

- Each wallpaper pattern has a group of symmetries. It turns out there are only 17 symmetry groups you can get this way! Illustrate the 17 wallpaper groups (by first drawing a wallpaper for each such possible symmetry type) and pick a few to describe the symmetry group in more detail.