Homework 3

1. Let G be the group of symmetries of a square in the 2-dimensional plane.
   (a) Analogous to what we did for the symmetries of an equilateral triangle, write down all symmetries of the square. What is the order of the group G?
   (b) Label the 4 vertices of the square with the numbers $1, 2, 3, 4$. Accounting for how these labels are moved around by each symmetry, write down a subgroup $H \subset S_4$ which corresponds to G.
   (c) Is H all of S_4? Is it contained in the alternating group A_4?
2. Compute the following compositions of permutations.
   (a) $(1345)(234)$
   (b) $(143)(23)(24)$
   (c) $(1354)^{100}$
3. (a) What is $\mathrm{ord}(\sigma)$ for $\sigma \in S_n$ equal to a cycle of length l? Prove your claim.
   (b) Recall that an arbitrary permutation $\sigma \in S_n$ can be written $\sigma = \sigma_1\sigma_2\cdots\sigma_k$ where each σ_i is a cycle and they are all disjoint. Show that
   $$\mathrm{ord}(\sigma) = \mathrm{lcm}(l_1, \ldots, l_k)$$
   where l_i is the length of the cycle σ_i.
   (c) Write down all possible orders of elements in S_7.
4. In lecture we saw that every cycle in S_n is a product of transpositions. Use this to explain how the parity of a cycle is determined by the length of the cycle. Then compute the parities of the following permutations:
   (a) (14356)
   (b) $(156)(234)$
   (c) $(17254)(1423)(154632)$
5. Find all of the subgroups in the alternating group A_4, and list their orders.
6. Show that A_{10} contains an element of order 15. Does A_{10} contain an element of order 14? Explain your answer.