PRACTICE PROBLEMS FOR EXAM 3

1. Let $f(x,y,z) = x+2y+z$ and let R be the solid $x^2+y^2+z^2 \le 4$, $\sqrt{3(x^2+y^2)} \le z$. Set up an iterated integral to compute $\iiint_R f(x,y,z)\,dxdydz$ (a) using rectangular coordinates (x,y,z), (b) using cylindrical coordinates (r,θ,z), and (c) using spherical coordinates (ρ,θ,φ). Do not evaluate the integrals!

2. Find the volume of the region in space bounded by the paraboloid $x = 1-y^2-z^2$ and the plane $x = 0$.

3. The solid E in the first octant is obtained by removing the cylinder $x^2+y^2 = 1$ from the sphere $x^2+y^2+z^2 = 4$. Set up a triple integral in cylindrical coordinates to compute the total mass of E if its density is given by $\rho(x,y,z) = z^2+\sqrt{x^2+y^2}$. Do not evaluate the integral.

4. (a) Find the volume of one of the wedges cut from the cylinder $x^2+y^2 = a^2$ by the planes $z = 0$ and $z = mx$, $m > 0$. (b) Use spherical coordinates to evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^2\,dz\,dy\,dx\,.$$

5. Let C be the portion of the helix $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + 4t\,\mathbf{k}$ between $t = 0$ and $t = \pi$, and let $\mathbf{F} = x\,\mathbf{i} + z\,\mathbf{k}$. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$.

6. Compute work done by vector field $\mathbf{F} = (-yx^2 + e^{x^2})\,\mathbf{i} + (xy^2 - e^{y^2})\,\mathbf{j}$ in moving a particle along the circle $x^2+y^2 = 1$ once in the counterclockwise direction.

7. Let $\mathbf{F} = ye^{xy}\,\mathbf{i} + (xe^{xy} - z^3)\,\mathbf{j} + (2\sin z - 3yz^2)\,\mathbf{k}$. (a) Evaluate curl F. (b) Find a function f such that $\mathbf{F} = \nabla f$. (c) Find the work done by $\mathbf{F}$ along the straight line segment from $(0,5,0)$ to $(1,0,\pi)$.

8. Find the work done by the force field $\mathbf{F}(x,y,z) = z\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$ in moving a particle from the point $(3,0,0)$ to the point $(0,\pi/2,3)$ (a) along a straight line and (b) along the helix $x = 3\cos t$, $y = t$, $z = 3\sin t$. Is this force field conservative? Justify your answer.

9. Let C_1 be the unit circle $x^2+y^2 = 1$ and C_2 the concentric circle of radius two. Orient both C_1 and C_2 counterclockwise. Suppose that $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j}$ is a vector field in plane such that

$$\int_{C_1} \mathbf{F} \cdot \mathbf{n}\, ds = 10 \quad \text{and} \quad \int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 17,$$

where $\mathbf{n}$ is the unit normal vector in said orientation.

(a) If $\mathbf{F}$ is continuous (together with its partial derivatives) in the plane, compute

$$\iint_D \operatorname{div} \mathbf{F}\, dA,$$

where D is the domain defined by the inequality $x^2 + y^2 \leq 1$.

(b) If $\mathbf{F}$ is continuous (together with its partial derivatives) on the annulus bounded by C_1 and C_2, and $Q_x = P_y$ everywhere on the annulus, compute

$$\int_{C_2} \mathbf{F} \cdot d\mathbf{r}.$$

10. Find the center of mass of the thin wire of density 1 bent into the shape of cycloid $\mathbf{r}(t) = (t - \sin t)\,\mathbf{i} + (1 - \cos t)\,\mathbf{j},\ \ 0 \leq t \leq 2\pi$.