PRACTICE PROBLEMS FOR EXAM 2

1. Let $f(x, y)$ have continuous second partial derivatives, and let $x = st$ and $y = e^{st}$.
(a) Find $\partial x/\partial t$ and $\partial y/\partial t$.
(b) Find $\partial f/\partial t$ in terms of $\partial f/\partial x$, $\partial f/\partial y$, s and t.
(c) Find $\partial^2 f/\partial t^2$ in terms of $\partial^2 f/\partial x^2$, $\partial^2 f/\partial x\partial y$, $\partial^2 f/\partial y^2$, $\partial f/\partial x$, $\partial f/\partial y$, s and t.

2. Consider the function $f(x, y) = 3x^2 - xy + y^3$.
(a) Find the rate of change of f at $(1, 2)$ in the direction of $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$.
(b) In what direction (unit vector) does f decrease at $(1, 2)$ at the maximum rate ? What is this maximum rate of change?
(c) In what directions is the rate of change of f at $(1, 2)$ equal to zero? Your answer should be a pair of opposite unit vectors.

3. Suppose the gradient $\nabla f(2, 4)$ of a function $f(x, y)$ has length equal to 5. Is there a direction $\mathbf{u}$ such that the directional derivative $D_{\mathbf{u}}f$ at the point $(2, 4)$ is 7? Explain your answer.

4. Find the tangent plane to the ellipsoid $x^2+4y^2 = 169-9z^2$ at the point $P = (3, 2, 4)$.

5. Find the points on the surface (ellipsoid) $x^2 + 2y^2 + 4z^2 + xy + 3yz = 1$ where the tangent plane is parallel to the xz plane.

6. Find all the critical points of $f(x, y) = x^2 + y^2/2 + x^2y$ and apply the second derivative test to each of them.

7. Find the absolute maximum and minimum values of the function $f(x, y) = (x - 1)^2 + (y - 1)^2$ in the rectangular domain $D = \{(x, y) : 0 \leq x \leq 1, \ \ 0 \leq y \leq 2\}$. Justify your answer.

8. Find the maximum of $f(x, y) = xy$ restricted to the curve $(x + 1)^2 + y^2 = 1$. Give both the coordinates of the point and the value of f.

9. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is a constant C.

10. Compute $\iint_D (3x + 1)\, dxdy$ where D is the region in the first quadrant bounded by the parabolas $y = x^2$ and $y = (x - 1)^2$ and the y–axis.

11. Change the order of integration in the following iterated integral:

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} f(x,y)\, dx\, dy.$$

12. (a) Find the area of the region enclosed by the cardioid given in polar coordinates by $r = 1 + \cos(\theta)$. (b) Use polar coordinates to evaluate

$$\int_{0}^{\sqrt{2}} \int_{y}^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy.$$