Vector operation rules

Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be any vectors. The cases you can have in mind are any vectors in n-dimensional Euclidean space $\mathbb{R}^n$. We write $\mathbf{0}$ for the zero vector. We have the following rules:

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$
$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$$
$$\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$$
$$\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = \mathbf{0}$$

Now let c, d, e be any scalars (real numbers). We have the following rules:

$$c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$$
$$(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$$
$$c(d\mathbf{u}) = (cd)\mathbf{u} = d(c\mathbf{u})$$
$$1\mathbf{u} = \mathbf{u}$$
$$0\mathbf{u} = \mathbf{0}$$

Note in the relation $0\mathbf{u} = \mathbf{0}$ the left hand side has the *scalar* 0 appearing, multiplying some vector $\mathbf{u}$, but on the right side of the equation is the zero *vector* $\mathbf{0}$ (in bold).

The operations of addition and scalar multiplication on vectors in $\mathbf{R}^n$ are defined as follows. Let $\mathbf{u} = (u_1, \ldots, u_n)$ and $\mathbf{v} = (v_1, \ldots, v_n)$. Let c be a scalar, i.e. a real number. Then

$$\mathbf{u} + \mathbf{v} = (u_1 + v_1, \ldots, u_n + v_n)$$
$$c\mathbf{u} = (cu_1, \ldots, cu_n)$$

Given these definitions you can directly verify all of the above rules.

Now suppose on our space of vectors we also have a *dot product*. This is an operation that takes two vectors $\mathbf{u}$ and $\mathbf{v}$ and produces a scalar. The rules for $\cdot$ are:

$$\mathbf{u} \cdot \mathbf{u} \geqslant 0, \text{ and } \mathbf{u} \cdot \mathbf{u} = 0 \text{ if and only if } \mathbf{u} = \mathbf{0}$$
$$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$$
$$(c\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$$
$$\mathbf{u} \cdot (c\mathbf{v} + d\mathbf{w}) = c(\mathbf{u} \cdot \mathbf{v}) + d(\mathbf{u} \cdot \mathbf{w})$$

The dot product in $\mathbb{R}^n$ is as follows. Let $\mathbf{u} = (u_1, \ldots, u_n)$ and $\mathbf{v} = (v_1, \ldots, v_n)$. Then:

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n$$

Given this definition you can directly verify all of the above rules.