Homework 8

In this homework you will explore determinants and eigenvalues/eigenvectors.

Problems in Strang:

§5.1: # 3, §6.1: # 6, 14, §6.2: # 2, 4

Do the following additional problems:

1. Compute the determinant of each matrix.

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 1 & 3 & 1 & 1 \end{bmatrix}$$

   Determine which matrices are invertible.

2. Recall that $\det(AB) = \det(A)\det(B)$ and also $\det(A^T) = \det(A)$. Show the following.
   (a) If A is invertible then $\det(A^{-1}) = 1/\det(A)$.
   (b) If Q is an orthogonal matrix, then $\det(Q) = \pm 1$.
   (c) An important collection of matrices, in math and physics, is the "special linear group":

$$SL(n, \mathbb{R}) = \{A \text{ an } n \times n \text{ real matrix}: \ \det(A) = 1\}$$

   Show that if A is in $SL(n, \mathbb{R})$ then so is the inverse A^{-1}. Show that if A and B are in $SL(n, \mathbb{R})$, then so is the product AB.

3. Find the eigenvalues and eigenvectors of the following matrices.

$$\begin{bmatrix} 3 & 5 \\ 1 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

   Using the information you've found, diagonalize each matrix. In other words, write each matrix A in the form $X\Lambda X^{-1}$. Compute the determinants of the matrices in two different ways: directly, and by using the expression $X\Lambda X^{-1}$.