Homework 5

In this homework you will explore the concepts of spanning, independence, and basis.

Problems in Strang:

§3.4: # 1, 2, 8, 11, 16, 18, 20

And the following problems:

A. Consider the following matrix, for a fixed real number θ:

$$A_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

As discussed in lecture, this matrix acts on vectors by rotating them by angle θ in the CCW direction. The fact that rotation is represented by a matrix has powerful consequences.

(i) For two different angles θ and ϕ, compute the product matrix $A_\phi A_\theta$. Argue geometrically that $A_\phi A_\theta$ should be the same matrix as $A_{\theta+\phi}$. Use this to derive formulas for $\cos(\theta+\phi)$ and $\sin(\theta+\phi)$. Takeaway: linear algebra recovers trigonometric identities!

(ii) Apply the same strategy to compute formulas for $\cos(3\theta)$ and $\sin(3\theta)$ for any given angle θ. (You will multiply three matrices this time.)

B. Suppose A is an $n \times n$ invertible matrix. Show that if $\mathbf{v}_1, \dots, \mathbf{v}_n$ is a basis of $\mathbb{R}^n$, then the set of vectors $A\mathbf{v}_1, \dots, A\mathbf{v}_n$ is also a basis of $\mathbb{R}^n$.

C. Let V be the vector space consisting of polynomials of degree at most n, which we considered in class. Show that the set of polynomials $1, x, x^2, \dots, x^n$ is a basis of V.