Homework 2

Problems in Strang:

§2.1: # 4, 9, 10, 17, 26

And the following problems:

A. (a) For any two vectors $\mathbf{a} = (a, b, c)$[1] and $\mathbf{a}' = (a', b', c')$ we define the cross product

$$\mathbf{a} \times \mathbf{a}' = (bc' - b'c,\ \ a'c - ac',\ \ ab' - a'b)$$

Use algebra to verify the identities $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{a}') = 0$ and $\mathbf{a}' \cdot (\mathbf{a} \times \mathbf{a}') = 0$. (Here $\cdot$ is the dot product.) It follows that $\mathbf{a} \times \mathbf{a}'$ is simultaneously perpendicular to $\mathbf{a}$ and $\mathbf{a}'$.

(b) Use the cross product to solve the following system of equations:

$$\begin{aligned} x + 2y - z &= 0 \\ 2x - y + 4z &= 0 \end{aligned}$$

(Hint: The solutions form a line. Look at what we did in class.)

B. (a) Consider the following system of linear equations:

$$\begin{aligned} x + 2y - z &= 0 \\ 2x - y + 4z &= 0 \\ x + y + cz &= -1 \end{aligned}$$

Here c is a constant. Solve for x, y, z in the case that $c = 1$. In this case the three planes intersect in a unique point – you should get one answer! (Use your solution to (B) for the first two planes. This is a line of the form $t(u, v, w)$. Plug into the third equation and solve for t.)

(b) Find the value of c for which the system has no solutions. In this case the third plane is parallel to the line of intersection of the first two planes.

[1] Sometimes we write vectors horizontally as (a, b, c). But we are just being temporarily lazy: our convention in the course (and in Strang) is that vectors in Euclidean space are column vectors.