# Homework 1

**Problems in Strang:**

§1.1: # 7, 9, 18, 19

**And the following problems:**

A. Consider vectors $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (1, 4)$. The set of vectors $t\mathbf{u} + (1 - t)\mathbf{v}$ where $t$ ranges over all real numbers forms a line; draw it. Do the same for $t(\mathbf{u} + \mathbf{v})$. Show that $(\mathbf{u} + \mathbf{v})/2$ is the intersection of these lines; explain the geometric meaning.

B. Recall that the dot product is related to the angle between vectors: $\mathbf{u} \cdot \mathbf{v} = \|u\|\|v\| \cos\theta$.

  (a) Compute the angle between $\mathbf{u} = (2, 1)$ and $\mathbf{v} = (-1, 5)$.[1]

  (b) Let $\mathbf{u}$, $\mathbf{v}$ be vectors in 10-dimensional space satisfying $\mathbf{u}\cdot\mathbf{u} = 10$, $\mathbf{v}\cdot\mathbf{v} = 5$ and $\mathbf{v}\cdot\mathbf{u} = -1$. Compute the angle between $\mathbf{u}$ and $\mathbf{v}$.

  (c) Let $\mathbf{u}$, $\mathbf{v}$ be vectors in 1000-dimensional space satisfying $\mathbf{u}\cdot\mathbf{u} = 1$, $\mathbf{v}\cdot\mathbf{v} = 2$ and $\mathbf{v}\cdot\mathbf{u} = 0$. Compute the angle between $\mathbf{u} + 2\mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

C. A line in $\mathbb{R}^2$ can be described by solutions $(x, y)$ to the equation $ax + by = c$, where $a, b, c$ are fixed scalars. This equation can also be written

$$\mathbf{a} \cdot \mathbf{x} = c$$

where $\mathbf{a} = (a, b)$ and $\mathbf{x} = (x, y)$.

  (a) Draw the 3 lines where $\mathbf{a} = (3, 2)$ and $c$ is in the set $\{-1, 0, 1\}$.

  (b) Let $\mathbf{a} = (a, b)$ and $c$ be arbitrary. Prove the line $\mathbf{a} \cdot \mathbf{x} = c$ is perpendicular to the line $t\mathbf{a}$ (the line described by all scalar multiples of $\mathbf{a}$).

[1] You can use a calculator to evaluate arccos, i.e. the inverse of cosine.