

# Bruno Benedetti

## Syllabus for the Topics Course “Polytopes”, Fall 2016

**Textbook:** G. M. Ziegler, *Lectures on Polytopes*, Springer.

**Content:** The course is based on chapters 0, 1, 2, 3, 4, 8, which I will integrate with material from recent papers. Tentative topics (and possible extra arguments for seminars) include:

- Definitions and examples; equivalence of vertex and facet description (without proof).
- Operations: Pyramids, Prisms, products, Minkowski sum, Connected Sum [p. 274].  
*Extras: Proof of equivalence theorem. Face vectors of cyclic polytopes are unimodal.*
- Caratheodory’s theorem [1.6]. Radon’s theorem [page 184].  
*Extra: Tverberg’s theorem.*
- Polarity [Chapter 2].
- Polytope graphs; The Hirsch conjecture [3.3].  
*Extras: Recent topics on Hirsch conjecture [2-3 seminar arguments available].*
- Balinski’s theorem [3.5]; Steinitz’ theorem [4.1]; Fary’s theorem [p.120]  
*Extras: Graphs of simple polytopes; Menger’s theorem and Max-Flow-Min-Cut; Embeddability of Simplicial Complexes.*
- Shellability [8.1 and 8.2].  
*Extras: Unshellable spheres.*
- h-vectors. Morse-theoretic interpretation and Dehn-Sommerville’s relations.
- Upper bound theorem, g-theorem, and g-conjecture [8.4, 8.5].

**Grading policy:** Attendance and interactive participation will play a key role. Students will have to present two seminars throughout the course. The presentation can be either at the whiteboard, or using an overhead projector; be aware that I will interrupt with questions and objections. The presentations are integrating part of the course, so the other students should attend too. There will be no final.

**General rules:** We will start from the basics; knowing topology helps, but all our polytopes will be subset  $\mathbb{R}^n$ , which is the most natural of all topological spaces. Questions, discussions, objections are always welcome. Cell phones are allowed, but must be put on silent mode. Same for pets.

**Office hours:** In Ungar 533 I will have office hours on Wednesday, from 10:30 to 12:30; but for this course I will keep an open door policy. So if you have questions, just come by!

Bruno Benedetti  
*Assistant Professor*