



MTH 785T: Polytopes

CLASS: TuTh 5:05PM - 6:20PM

Textbook: G. Ziegler, *Lectures on Polytopes*, integrated with some more recent material.

Content: Chapters 0–3, 4, 8, integrated with recent material. Tentative topics list, with possible arguments for seminars:

1. Definitions. Operations (pyramids, prisms, products, Minkowski sums, connected sums [p. 274]...).
Extras: From vertex to facet description. Associahedra and permutahedra. Cyclic polytopes are unimodal.
2. Convexity: Caratheodory [Ch. 1.6] and Radon's theorem [p. 184].
Extras: Tverberg's theorem. Colorful versions.
3. Polarity [Chapter 2].
Extras: The 24-cell. The dual block complex.
4. Polytope graphs. Simplex method and Hirsch conjecture [3.3]. Balinski [3.5], Steinitz [4.1], Fary's theorem [p.120].
Extras: Progress on Hirsch conjecture. Menger and Max-Flow-Min-Cut. Embeddability of complexes.
5. Shellability [8.1 and 8.2].
Extras: Unshellable spheres. The number of spheres.
6. h-vectors. Morse-theoretic interpretation and Dehn–Sommerville relations. g-theorem and g-conjecture [8.4, 8.5].
Extras: Generalized lower bound theorem.

Grading policy:

- Students will have to present at least two seminars throughout the course. The presentation can be either at the whiteboard, or using an overhead projector; be aware that I will tediously interrupt with questions and objections, partly because it's my job, partly because it's my character.
- Attendance and interactive participation will play a key role. The presentations are integrating part of the course, so the other students should attend too. There will be no final.

General rules:

- We will start from the **basics**; knowing topology helps, but all our polytopes will be subset of \mathbb{R}^n .
- All students are bound by the Academic Integrity Policy, which is intended to promote and support an atmosphere conducive to learning. Each student's work is to be their own unless express permission is given by the instructor. Additionally, while students are bound by the policy, faculty are also bound to report violations as offered in the policy.
- Cell phones are allowed in class, but must be on silent mode. Same policy for pets and lovers.

With this, I wish you a lovely Spring semester here at The U!

Bruno Benedetti
Associate Professor