UNIVERSITY
OF MIAMI

Dr. Bruno Benedetti
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Office hours: Wed 15:30–17:30
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MTH 785Q: Polytopes

Class: TuTh 12:30PM - 1:45PM

Textbook: G. Ziegler, *Lectures on Polytopes*, integrated with some more recent materia; plus the first chapter of Lakatos' *Proofs and refutations*, available at `https://math.berkeley.edu/~kpmann/Lakatos.pdf`.

Content: Chapters 0–3, 4, 8, integrated with recent material. Tentative topics list, with possible arguments for seminars:

1. Definitions. Operations (pyramids, prisms, products, Minkowski sums, connected sums [p. 274]...).
   Extras: From vertex to facet description. Associahedra and permutahedra. Cyclic polytopes are unimodal.
2. Convexity: Caratheodory [Ch. 1.6] and Radon's theorem [p. 184].
   Extras: Tverberg's theorem. Colorful versions.
3. Polarity [Chapter 2].
   Extras: The 24-cell. The dual block complex.
4. Polytope graphs. Simplex method and Hirsch conjecture [3.3]. Balinski [3.5], Steinitz [4.1], Fary's theorem [p.120].
   Extras: Progress on Hirsch conjecture. Menger and Max-Flow-Min-Cut. Embeddability of complexes.
5. Shellability [8.1 and 8.2].
   Extras: Unshellable spheres. The number of spheres.
6. h-vectors. Morse-theoretic interpretation and Dehn–Sommerville relations. g-theorem and g-conjecture [8.4, 8.5].
   Extras: Generalized lower bound theorem.

Grading policy:

- Students will have to present at least two seminars throughout the course. The presentation can be either at the whiteboard, or using an overhead projector; be aware that I will tediously interrupt with questions and objections, partly because it's my job, partly because it's my character.
- Attendance and interactive participation will play a key role. The presentations are integrating part of the course, so the other students should attend too. There will be no final.

General rules:

- We will start from the **basics**; knowing topology helps, but all our polytopes will be subset of $\mathbb{R}^n$.
- Cell phones are allowed in class, but must be on silent mode. Same policy for pets, spouses, and lovers.

Remarks:
We won't have classes in the September 17 week, I'm at a conference. (We'll make up for it during the course.)

With this, I wish you a lovely Fall semester here at The U!

Bruno Benedetti
Assistant Professor