Math 309: Second Midterm

Justify all steps for full credit. Throughout, n, k are positive integers, with $n \geq 3$.

Name:

Problem 1.[vertex degrees] Let G be the graph of vertices $1, 2, 3, 4$ and edges $12, 13, 14, 23, 24, 34$. Can it be drawn starting from some vertex but without ever lifting the pen from the paper?

Problem 2[Trees] Let G be an acyclic graph with n vertices and exactly k connected components. Show that G has $n - k$ edges.

Hint: Start by proving that every connected component is a tree.

Problem 3.[Planarity] Draw an example of a graph that is

- planar and 2-connected
- planar, but not 2-connected
- 2-connected, but not planar
- (BONUS POINTS) neither planar, nor 2-connected.

Problem 4.[Menger] Let G be a k-connected graph with n vertices. Prove that between any two non-adjacent vertices a and b of G, there is a path consisting of not more than $\frac{n-2}{k} + 1$ edges.

Hint: Use Menger. By contradiction, if you have many vertex-disjoint, long paths between a and b, then many vertices are touched altogether by all these paths! But G has only n vertices...

Problem 5[Ford–Fulkerson]. Find the maximum flow value in the following (s,t)-network, where the edge capacities have been marked in black.

Bonus points: (1) Indicate the cut of minimum capacity. (2) Explain why it's not a surprise that the maximum flow is integral, even if not all capacities are integers.