Music of the Spheres

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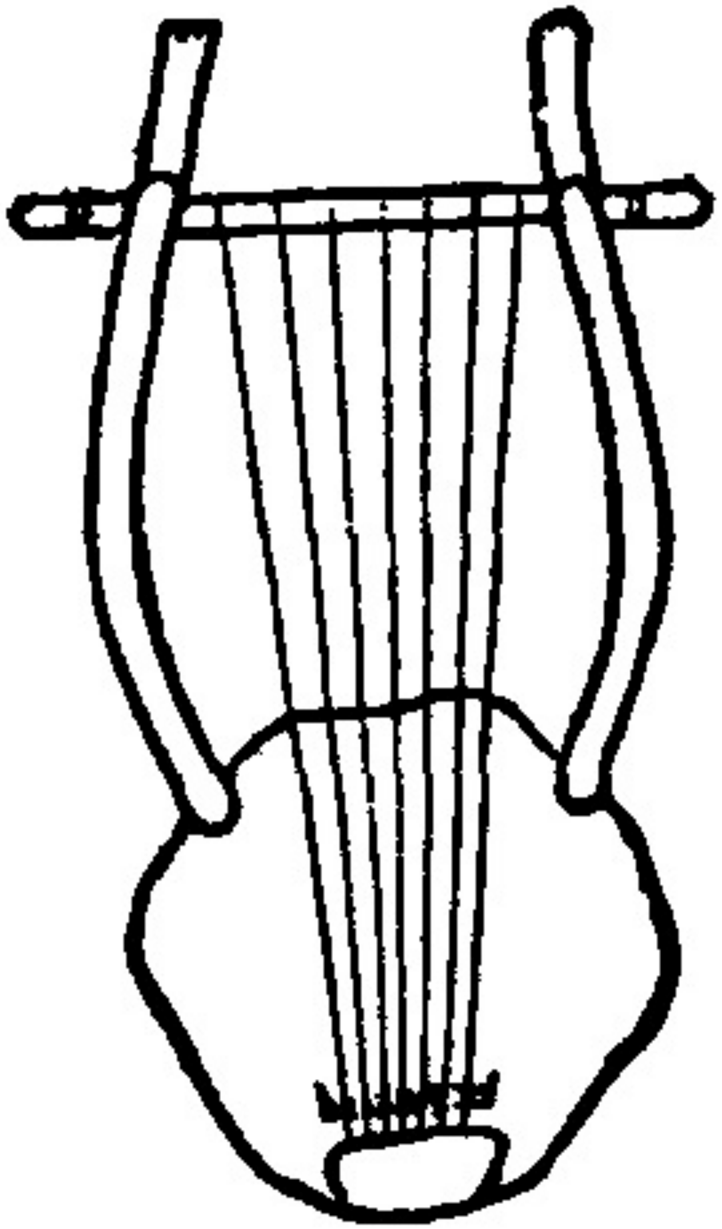
University of Miami

February 2, 2013

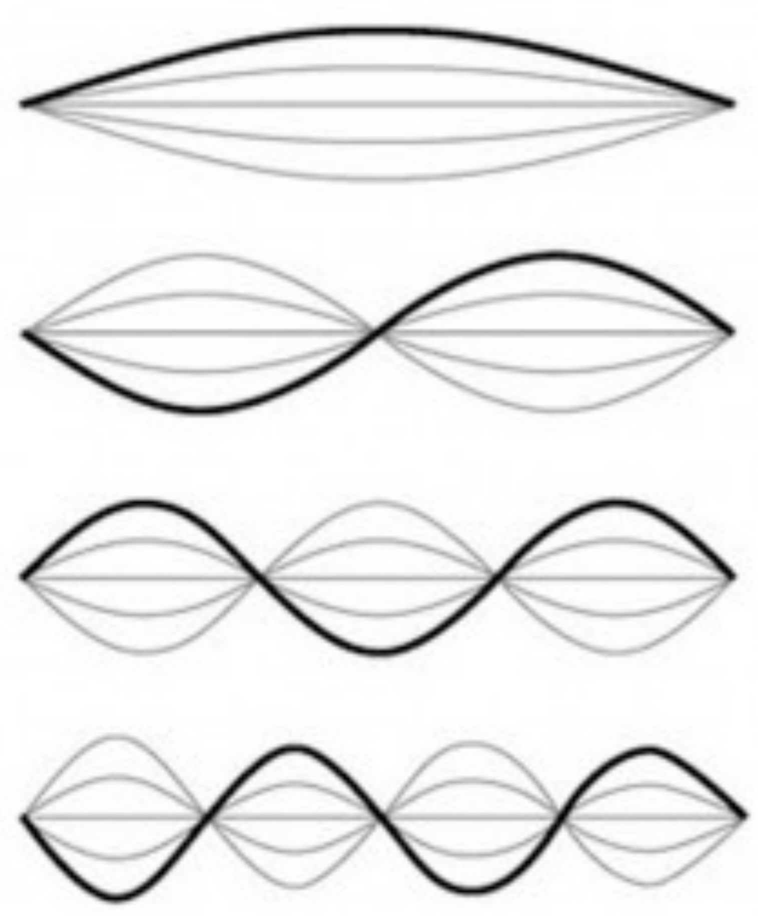
The Plucked String



The Plucked String

A plucked string with fixed ends can only vibrate at

certain resonant frequencies



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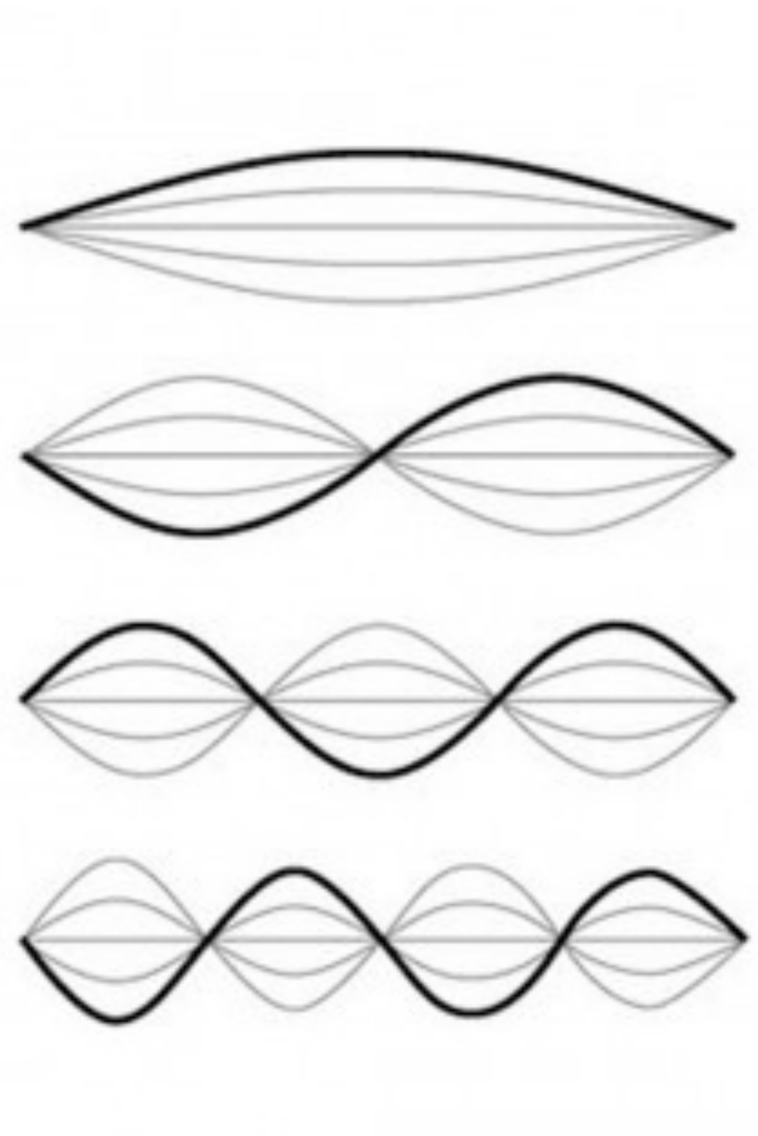

f "fundamental" frequency

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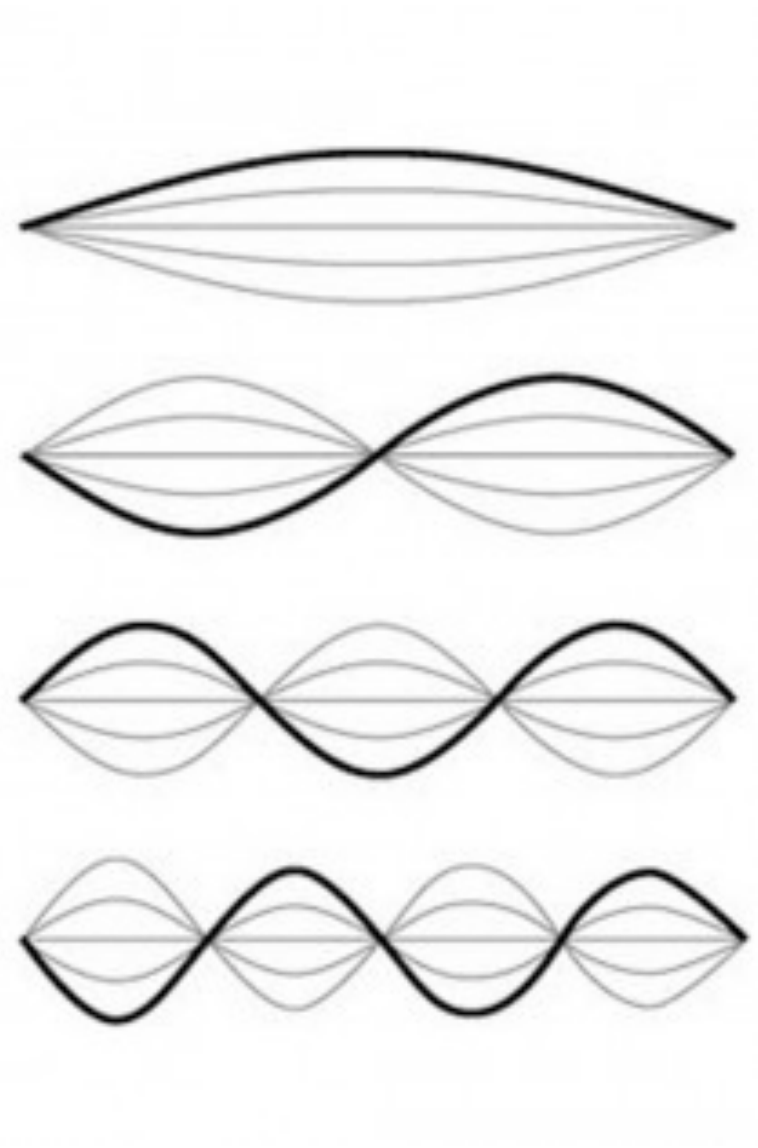

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$2f$ 1st harmonic

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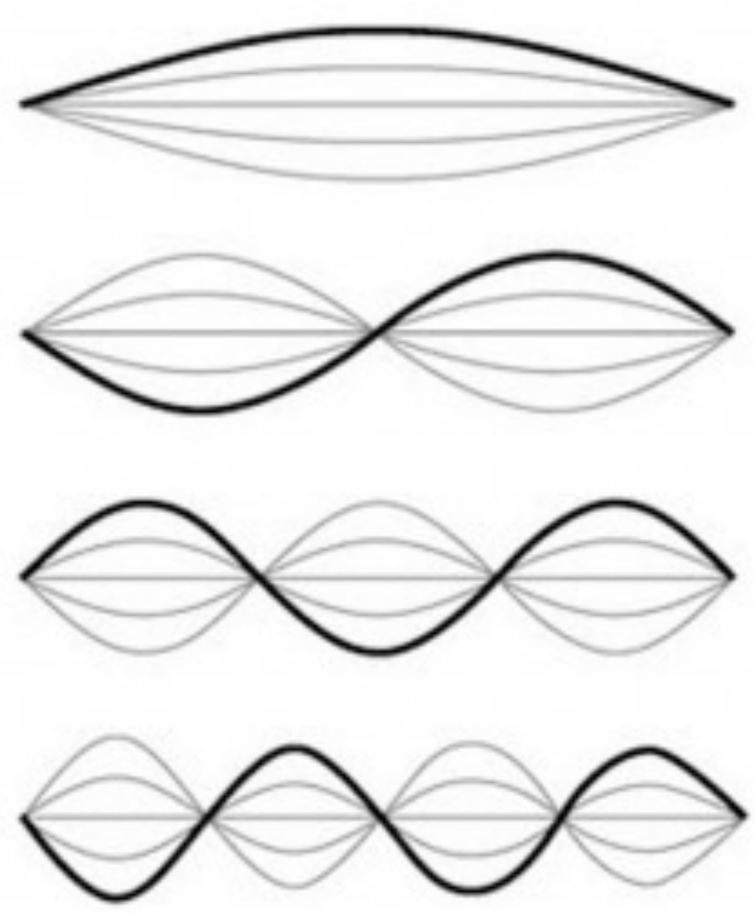

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$2f$ 1st harmonic

$3f$ 2nd harmonic

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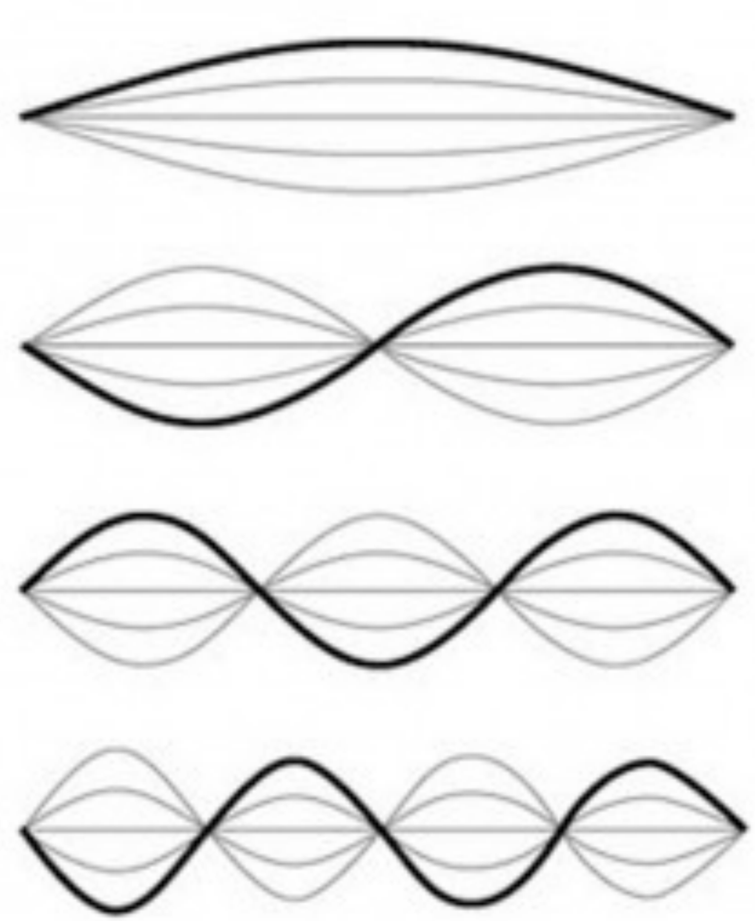
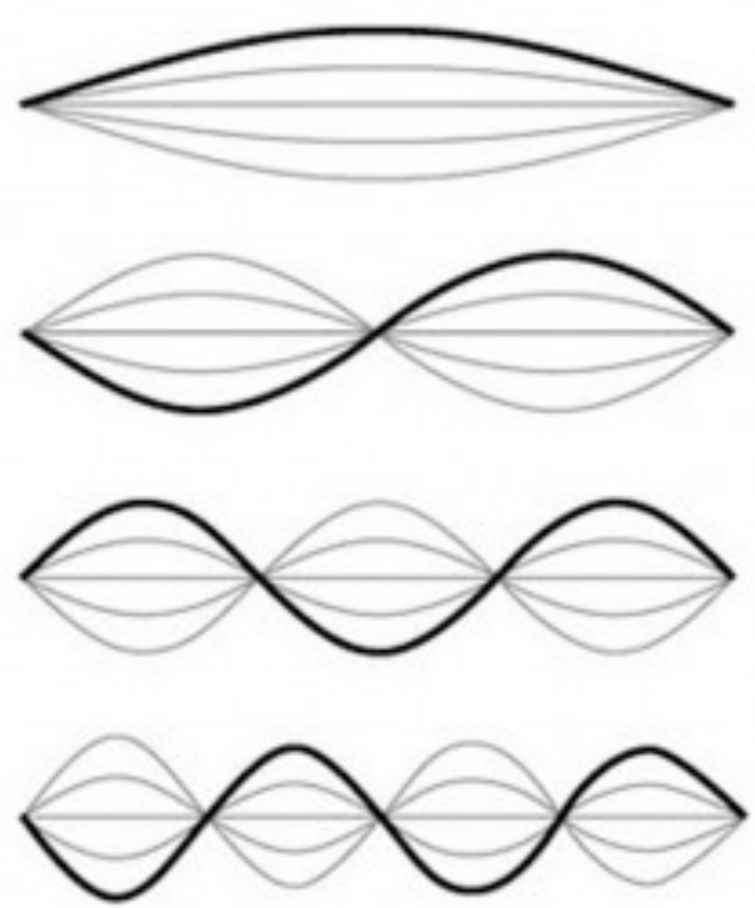


f	"fundamental" frequency
$2f$	1st harmonic
$3f$	2nd harmonic
$4f$	3rd harmonic

The Plucked String

Every possible vibration is a “superposition” of these.



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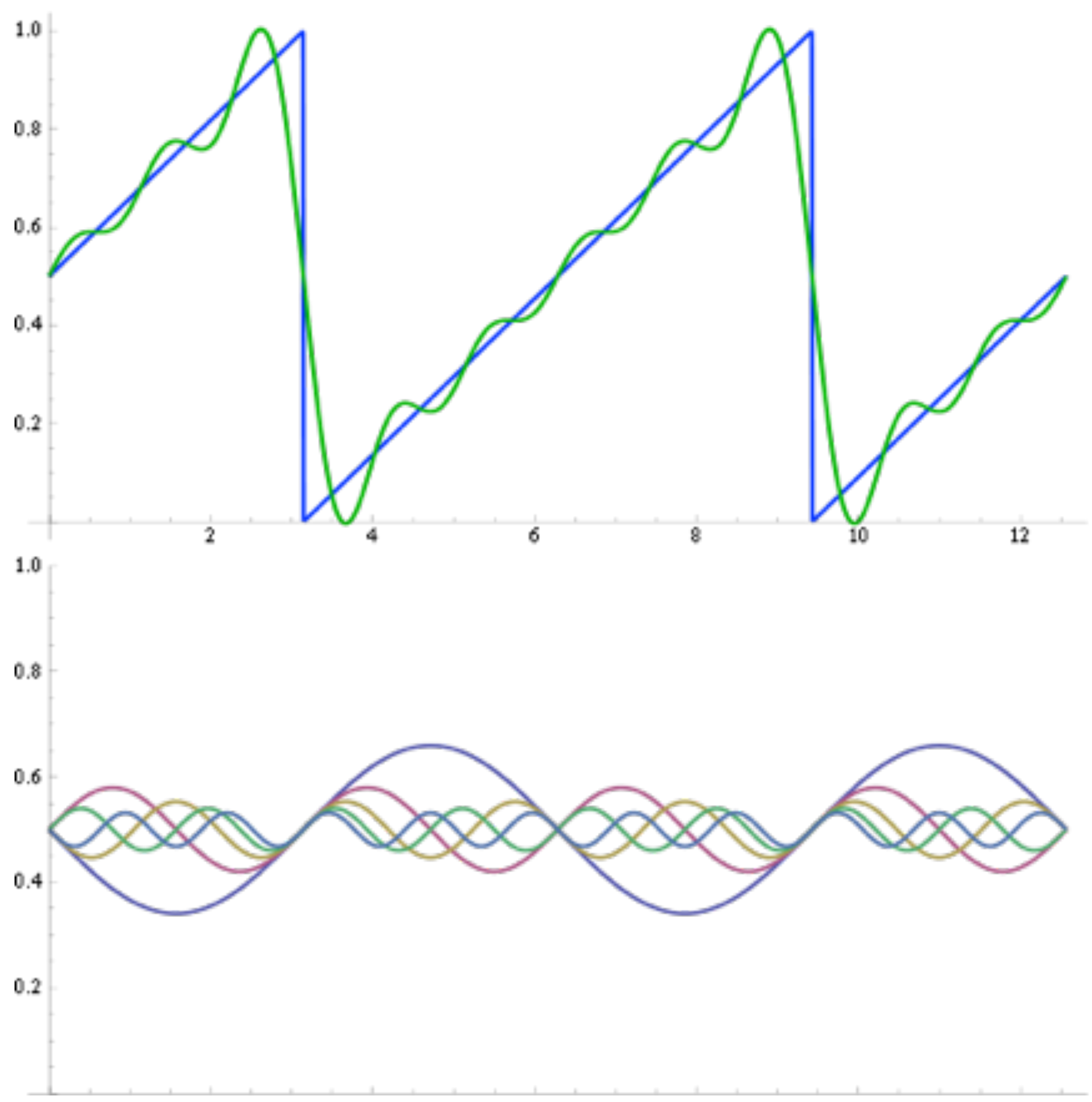

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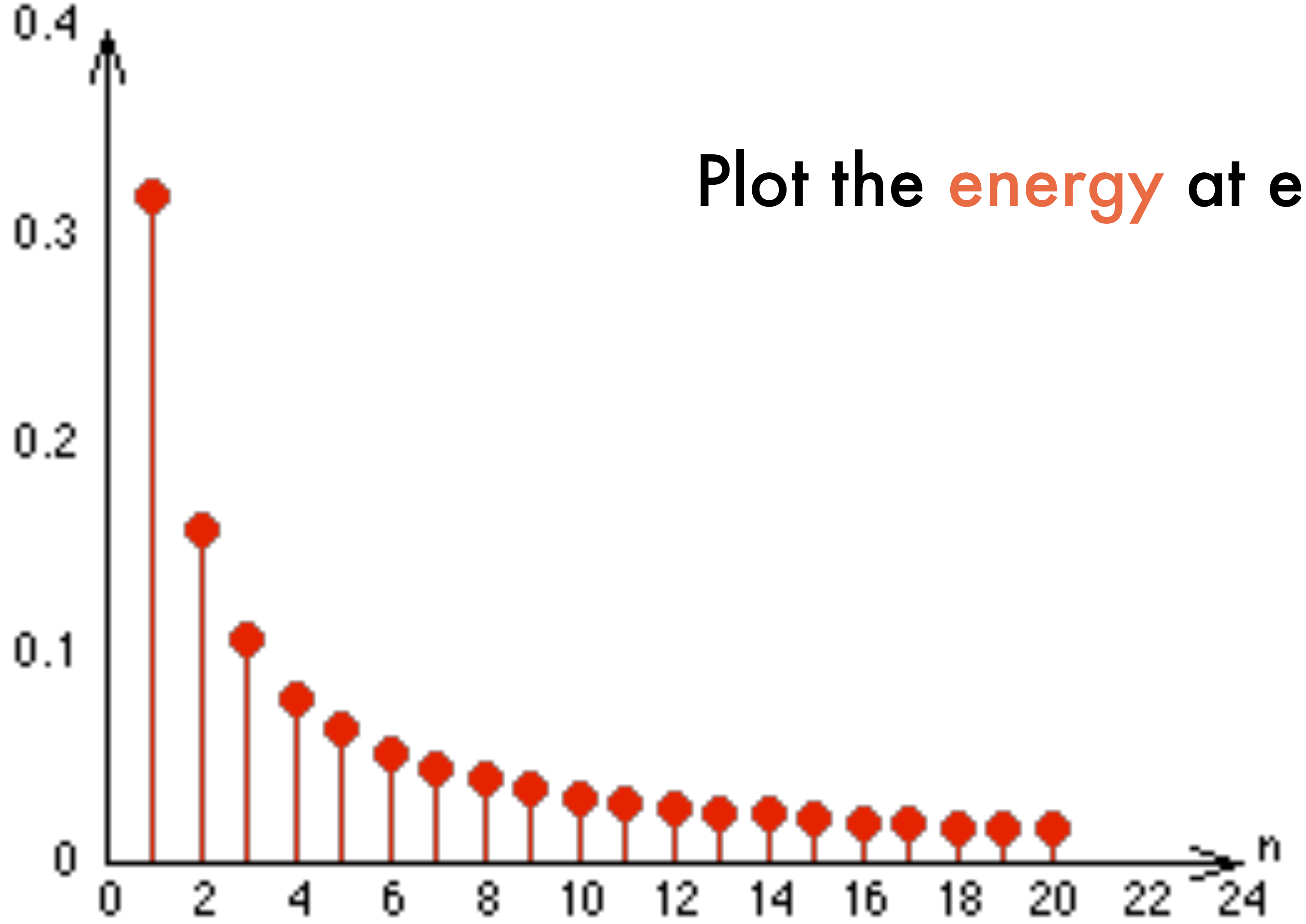
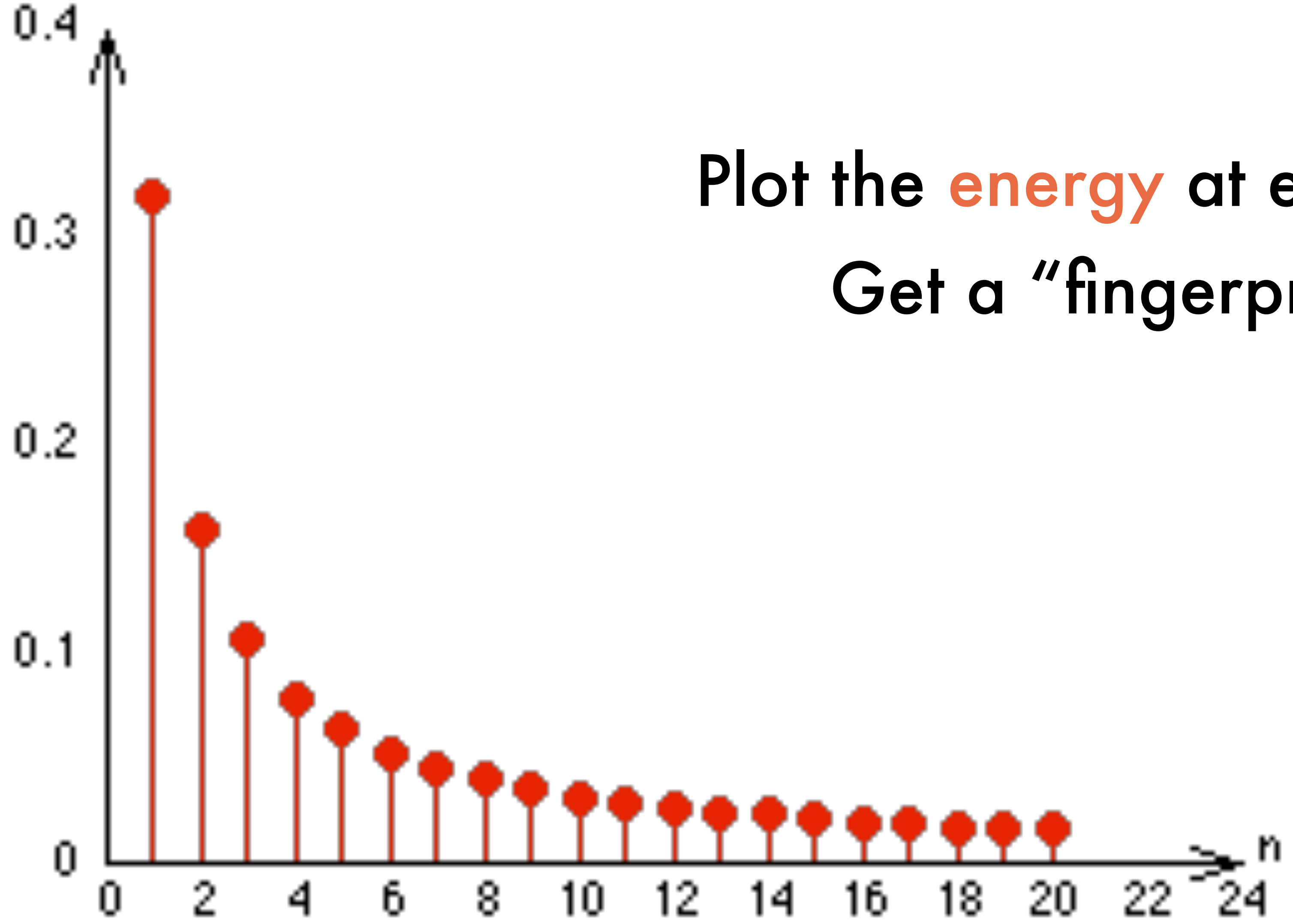


Plot the energy at each frequency:

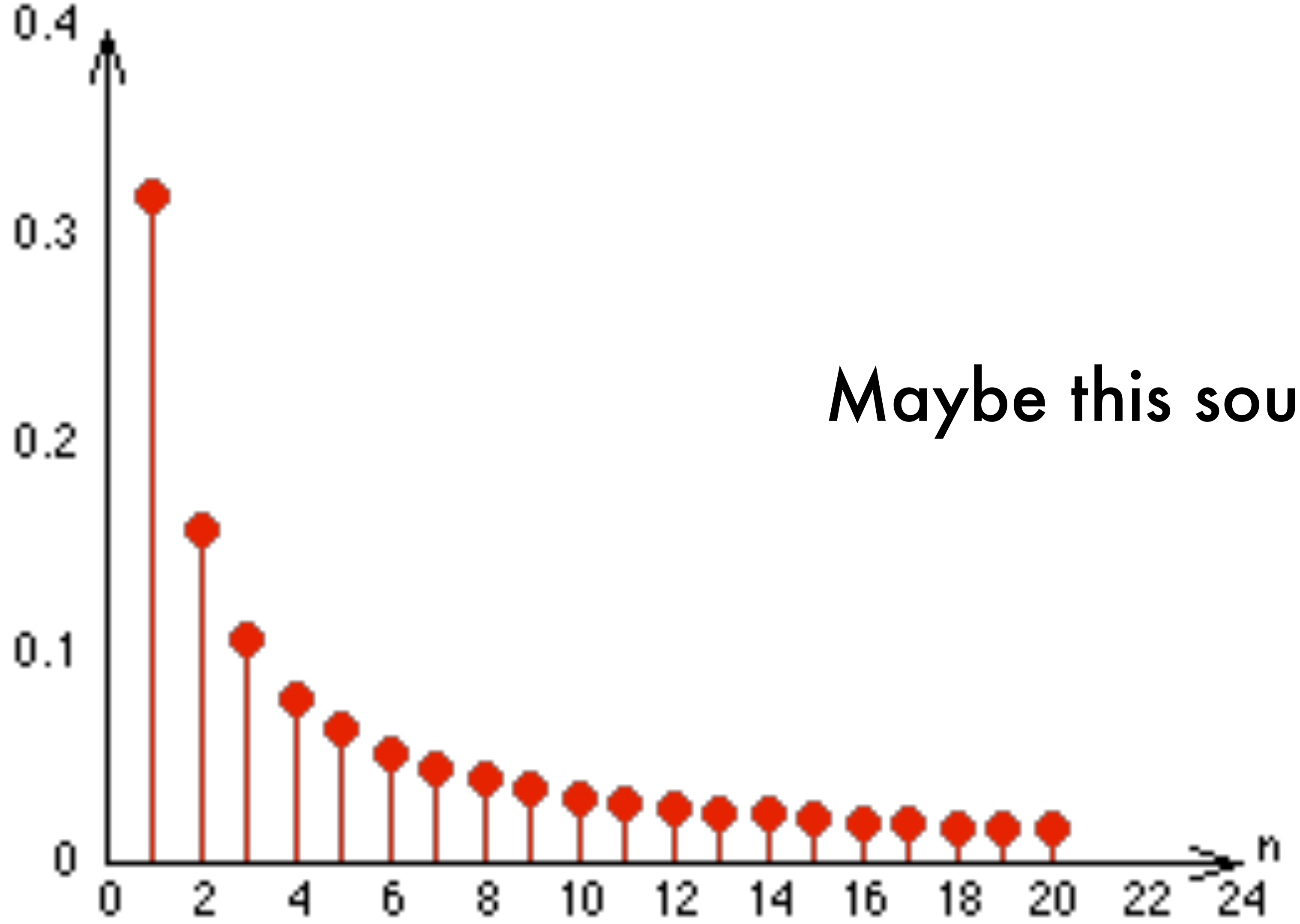
The Plucked String



Plot the energy at each frequency:

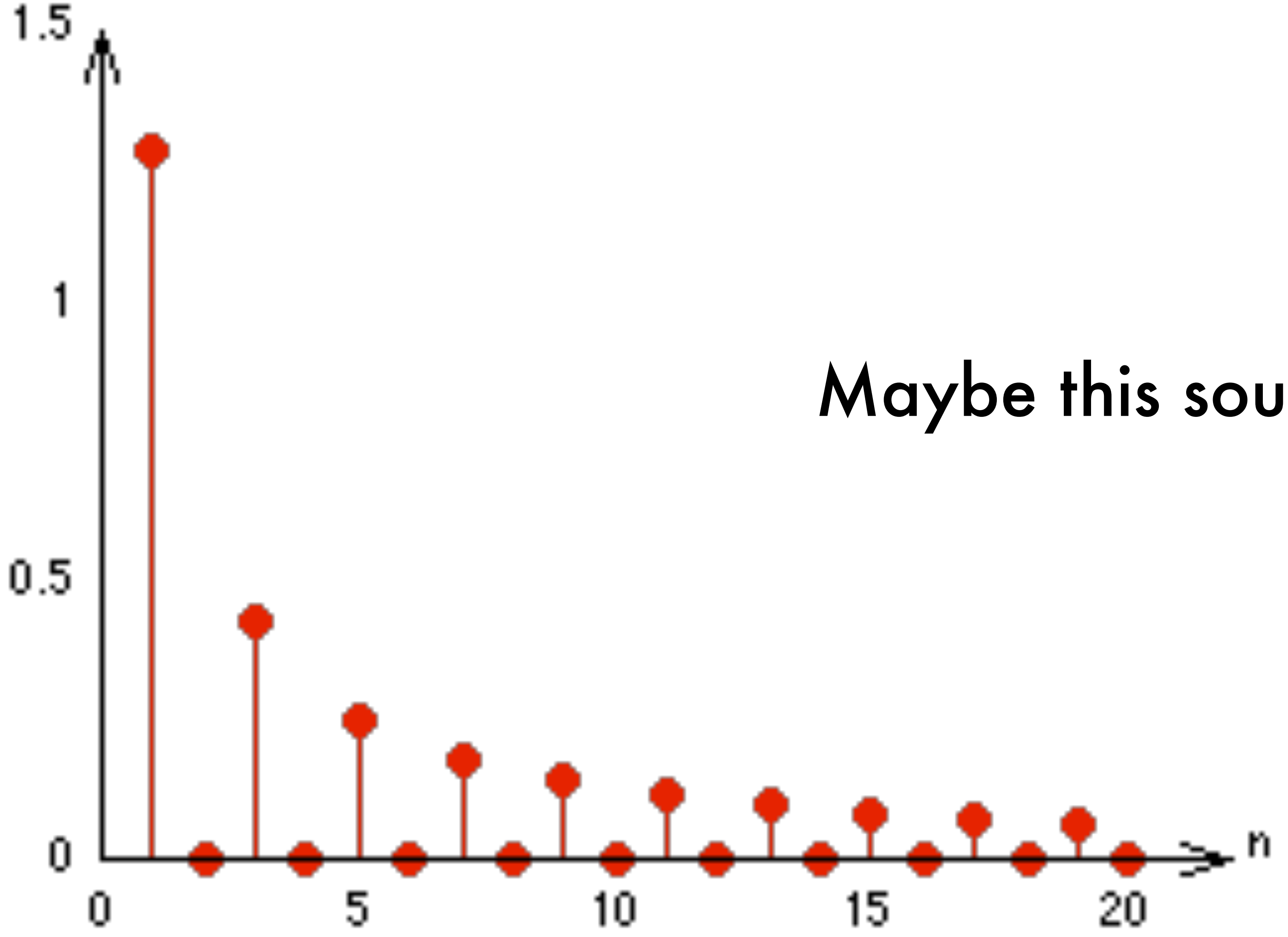
Get a “fingerprint” for the sound.

The Plucked String



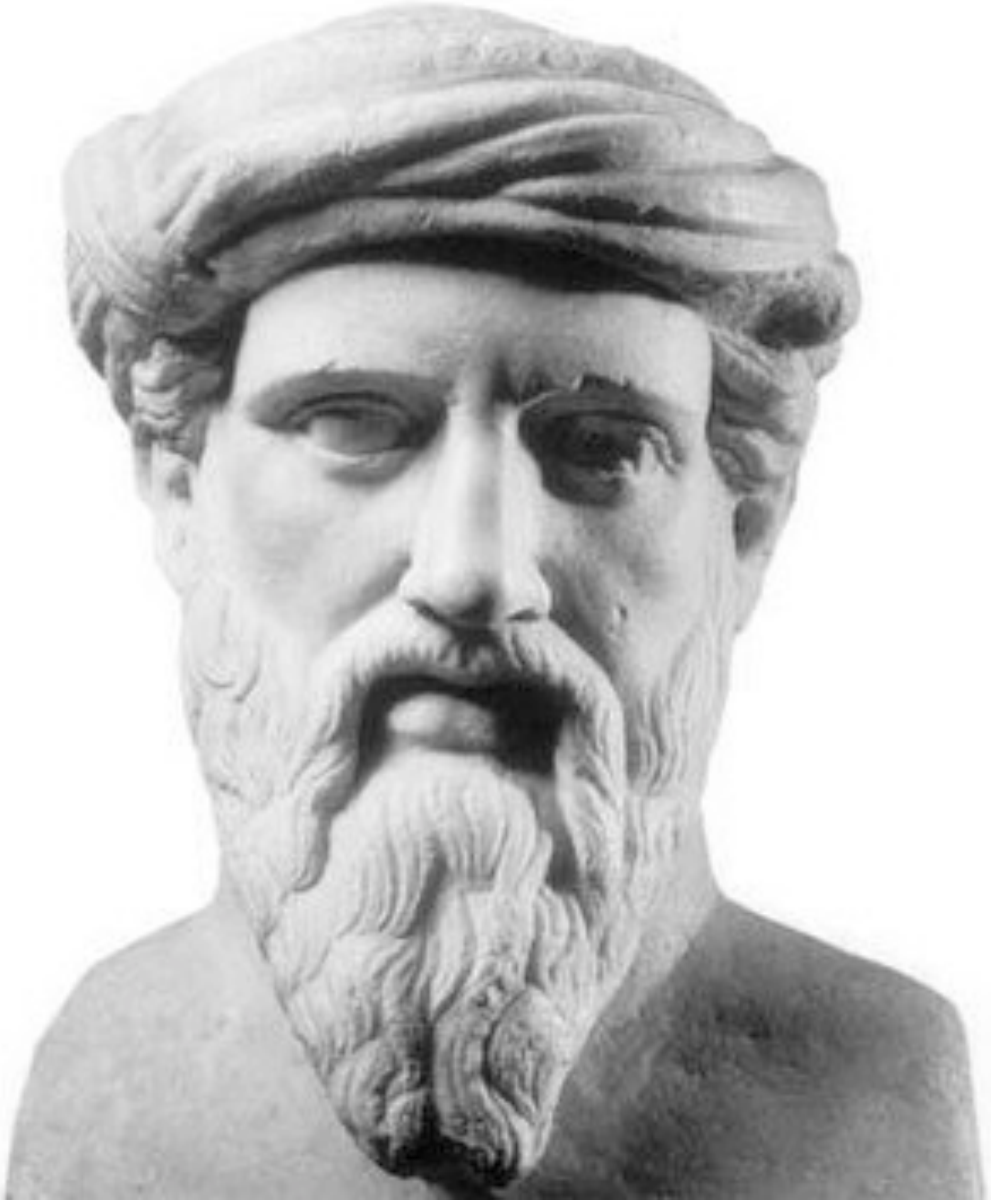
Maybe this sounds like a VIOLIN

The Plucked String



Maybe this sounds like a CLARINET

Pythagoras (c. 570 – 495 BC)



Pythagoras (c. 570 – 495 BC)

Big Discovery:

- When two strings of equal tension are plucked,
- They sound good if their lengths have the ratio of small whole numbers.

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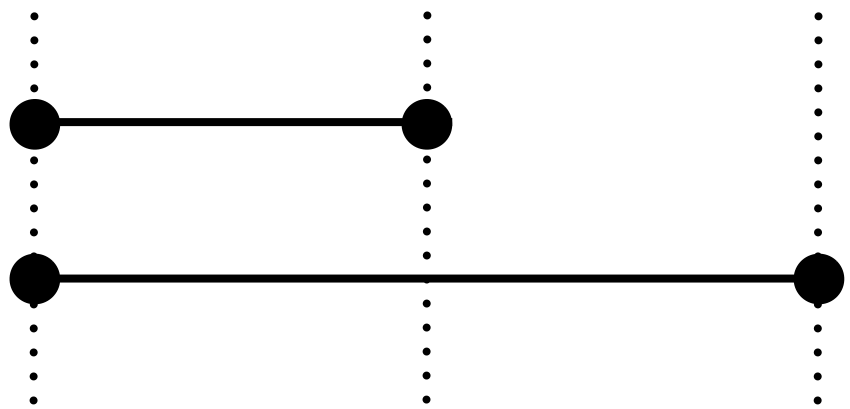
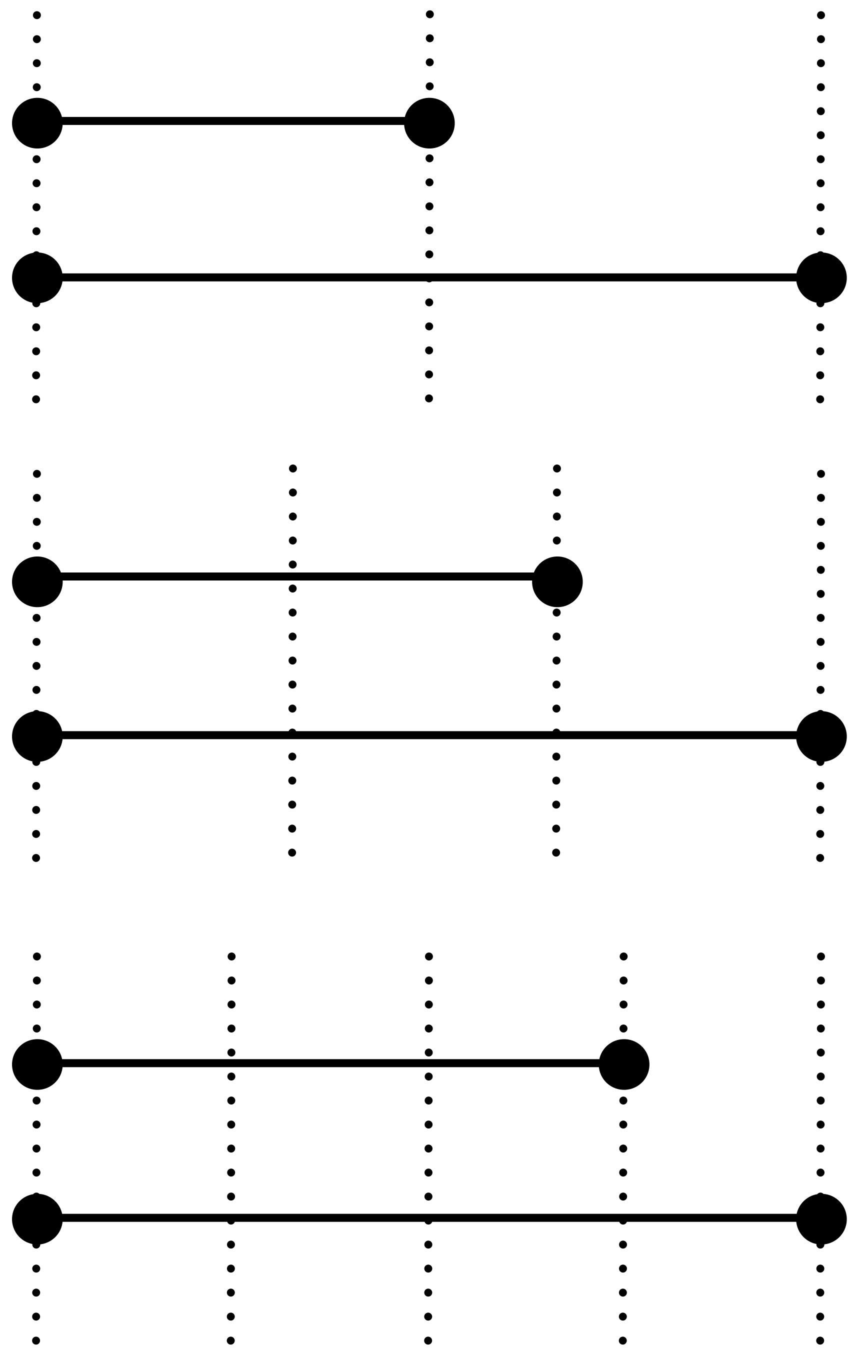


Eureka!

Pythagoras (c. 570 – 495 BC)



this sounds very good

("octave")

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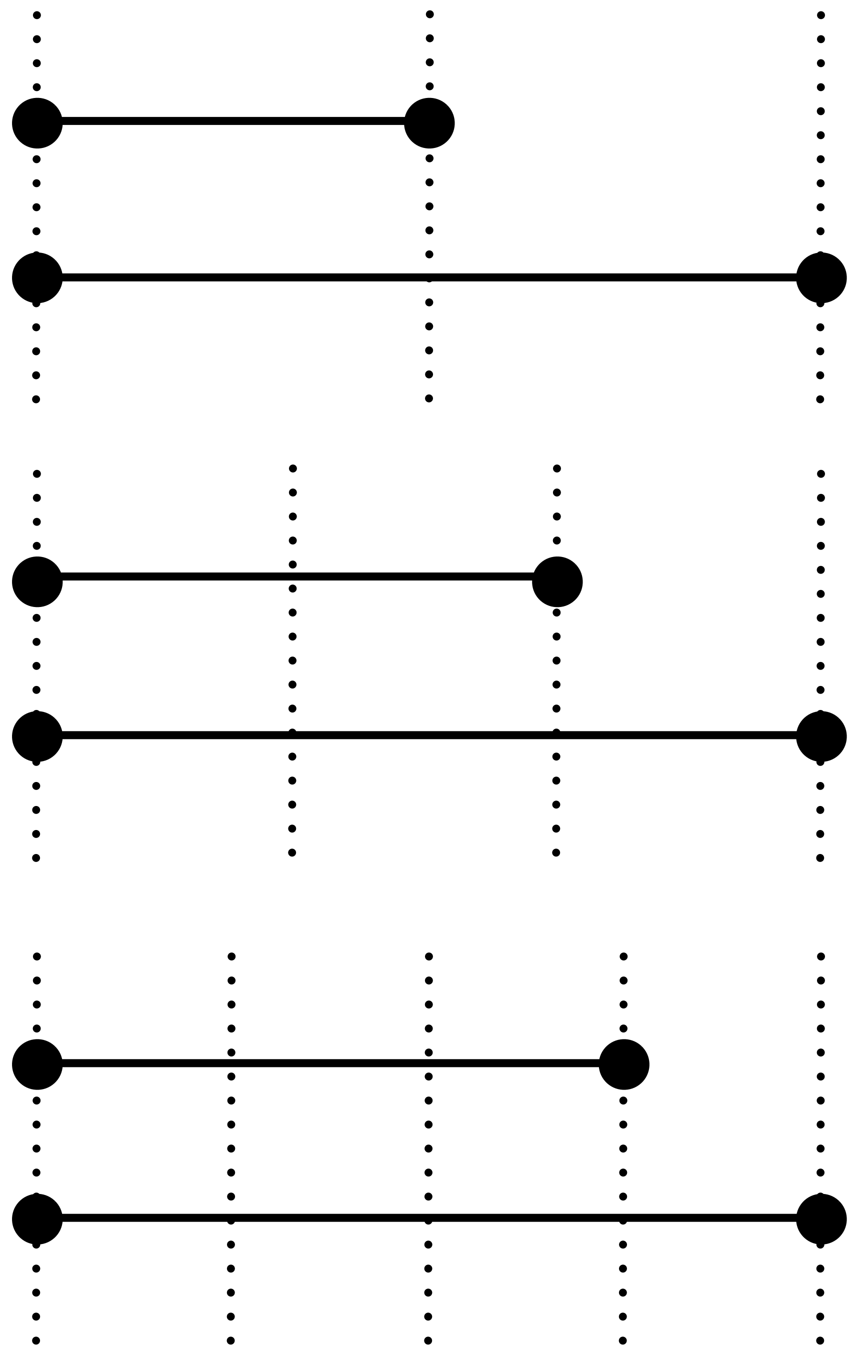

this sounds very good
("octave")

this sounds quite good
("perfect fifth")

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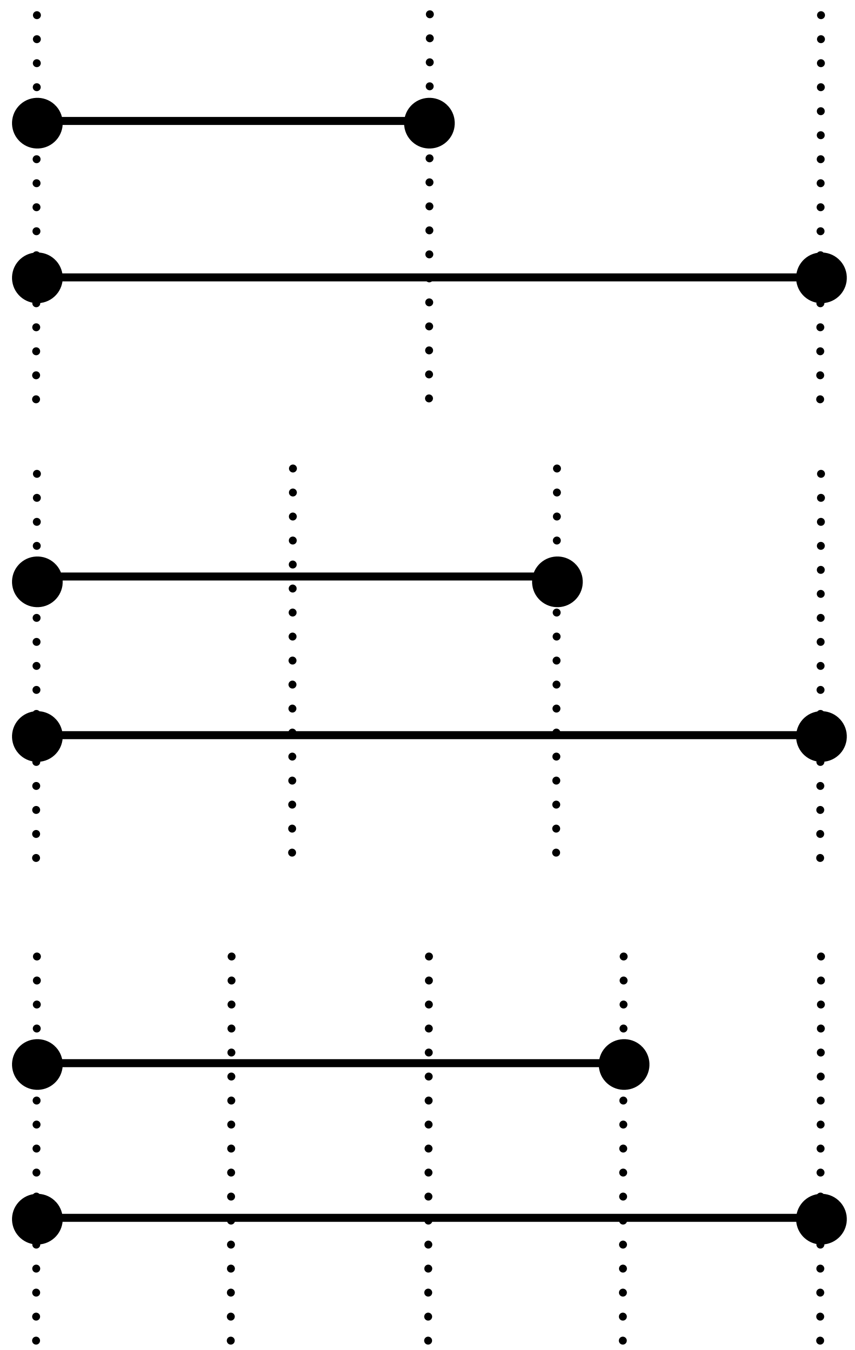


this sounds very good

("octave")

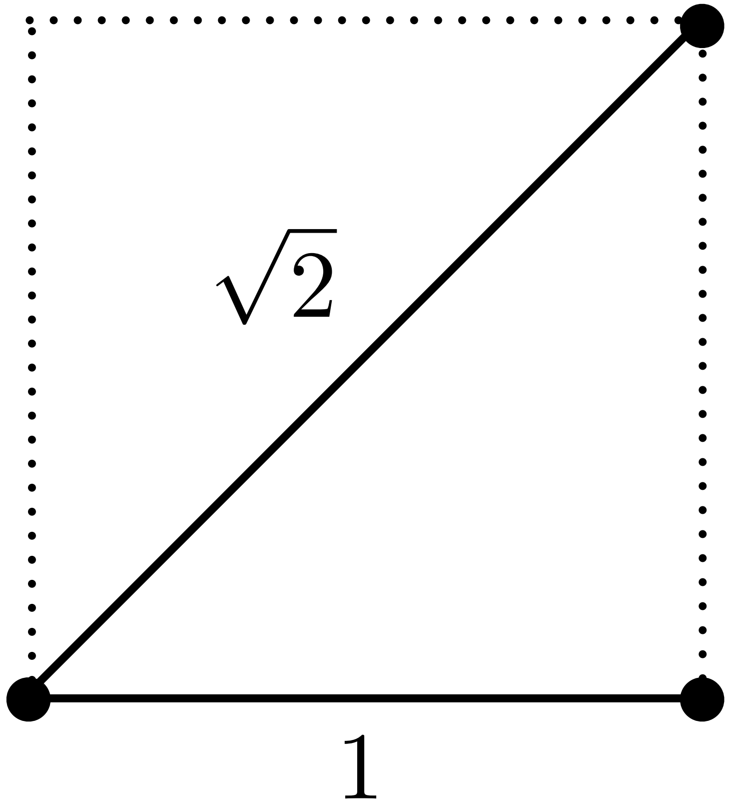
this sounds quite good

("perfect fifth")

this sounds rather good

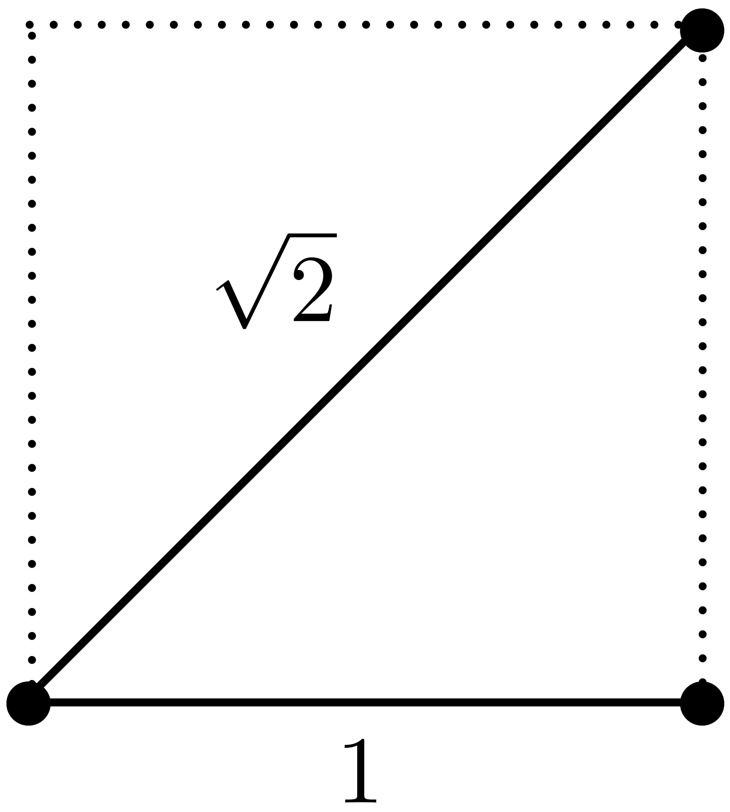
("perfect fourth")

Pythagoras (c. 570 – 495 BC)



this sounds TERRIBLE!

Pythagoras (c. 570 – 495 BC)



this sounds TERRIBLE!

$\sqrt{2}$ is not a fraction of whole numbers.

Modern Explanation

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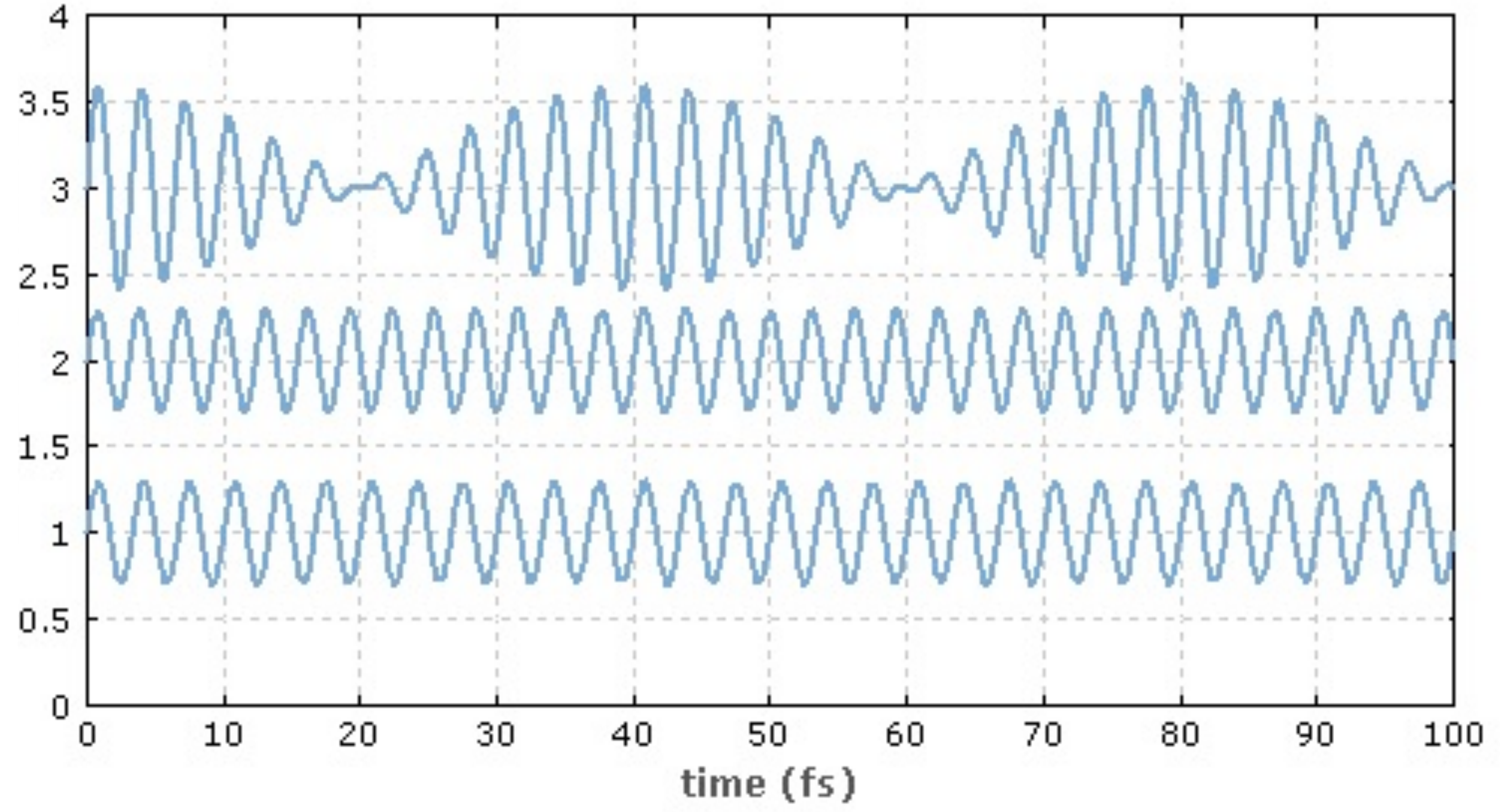
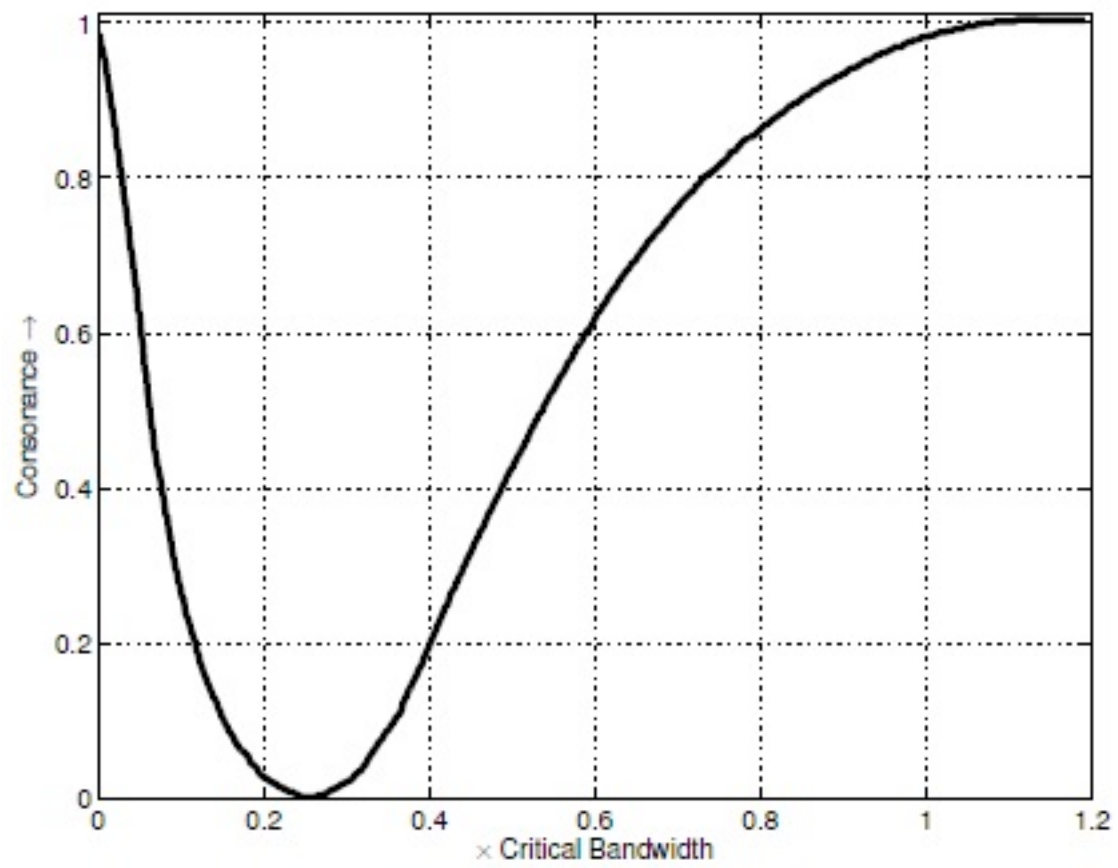


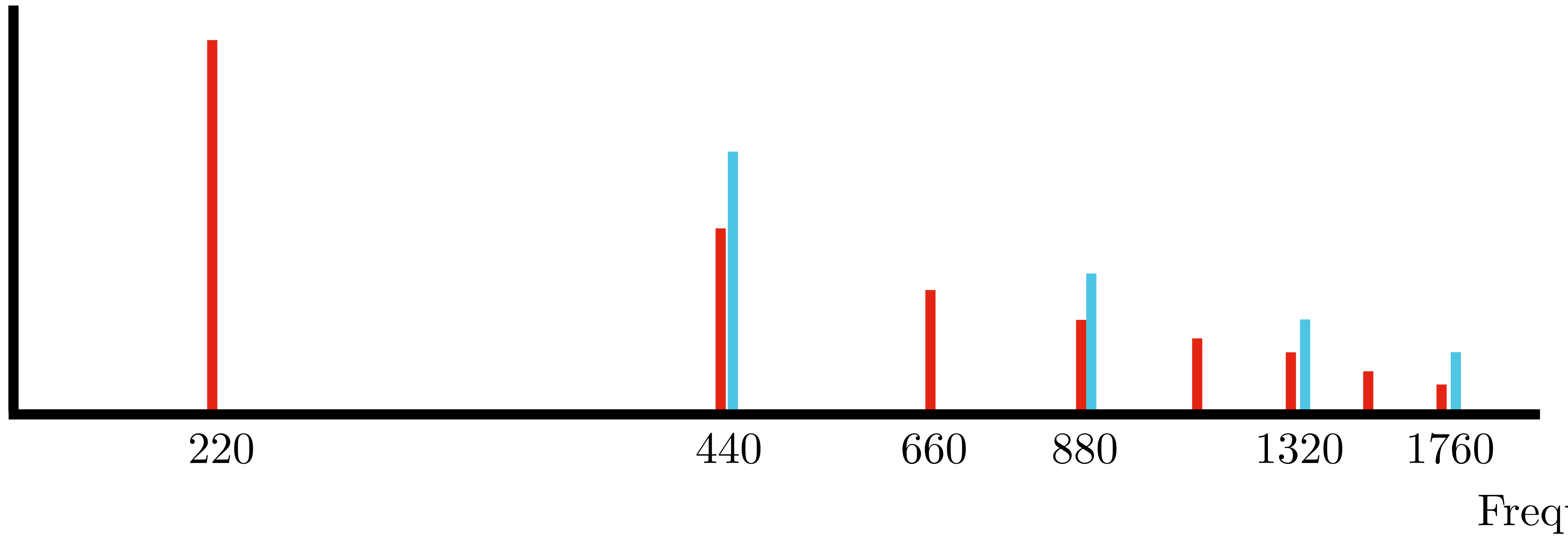
Two nearby frequencies sound ROUGH together. ("beats")

Modern Explanation



Plomp and Levelt (1965) measured the psychological effect.

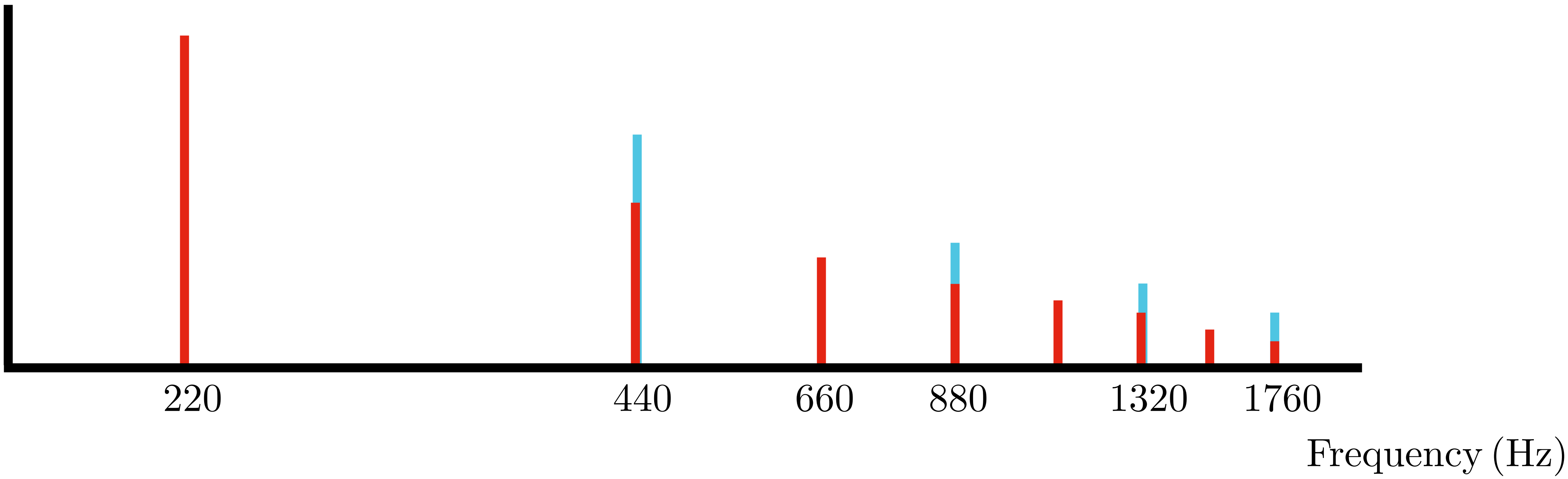
Modern Explanation



Play the notes 220 Hz and 445 Hz together: ROUGH

	445		890		1335		1780
220	440	660	880	1100	1320	1540	1760

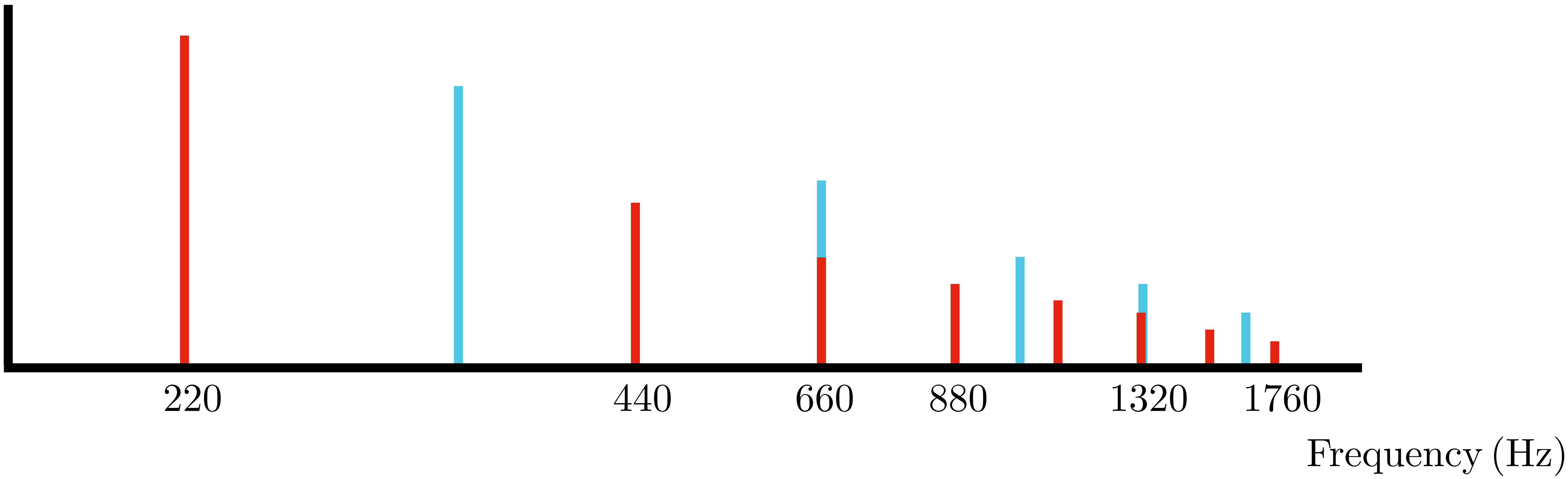
Modern Explanation



Play the notes 220 Hz and 440 Hz together: SMOOTH

	440		880		1320		1760
220	440	660	880	1100	1320	1540	1760

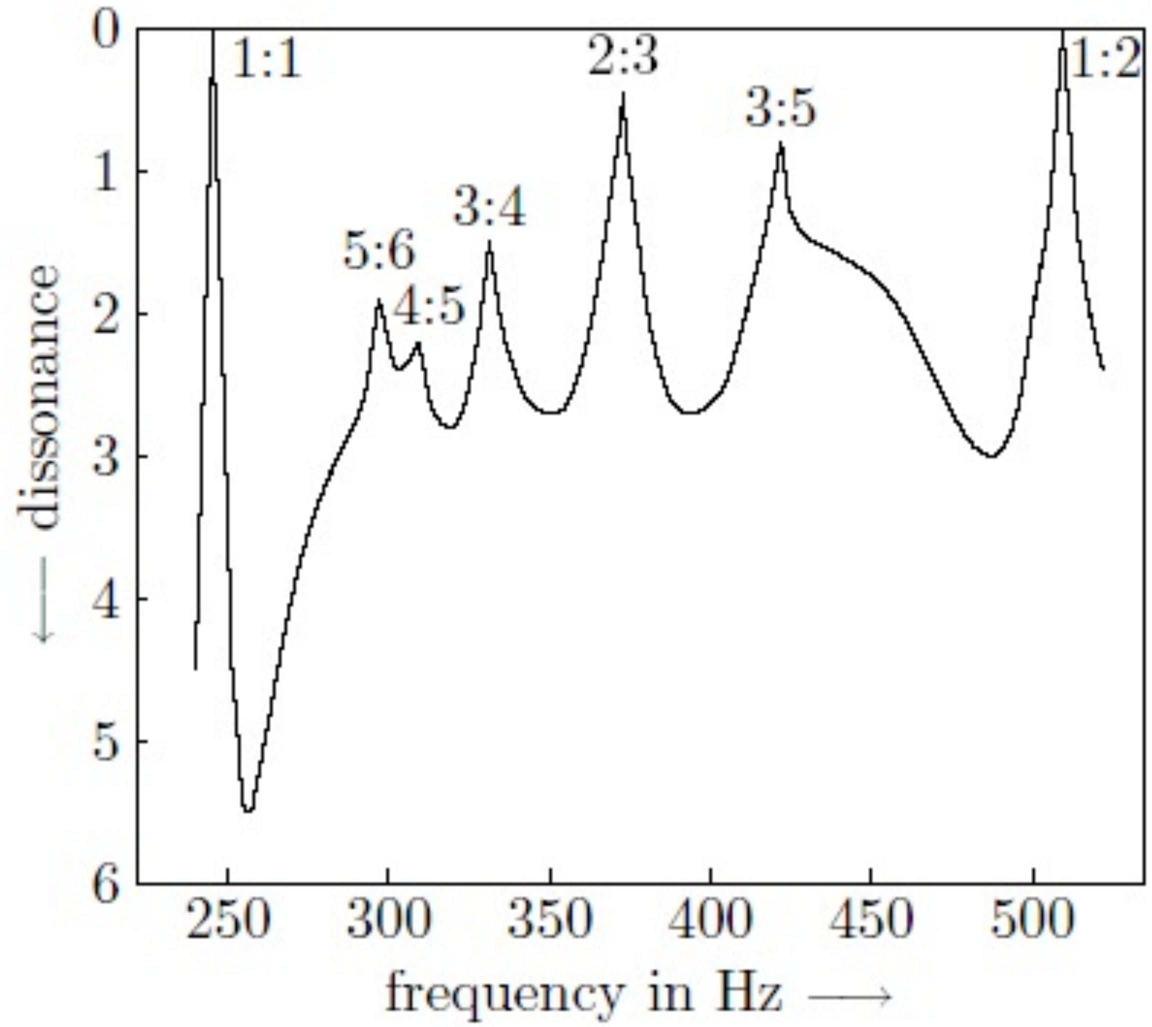
Modern Explanation



Play the notes 220 Hz and 330 Hz together: STILL NICE

330	660	990	1320	1650
220 440	660	880 1100	1320	1540 1760

Modern Explanation



The Plomp-Levelt curve based on six harmonics.

Why do we have 12 notes?

Why do we have 12 notes?

Pythagorean tuning is based on the most consonant interval:

2:3

(a “perfect fifth”)

Why do we have 12 notes?

Two successive perfect fifths equals

$$\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$$

Why do we have 12 notes?

Three successive perfect fifths equals

$$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

Why do we have 12 notes?

Many successive perfect fifths equals

$$\frac{2}{3} \cdot \cdots \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{2^n}{3^n}$$

Why do we have 12 notes?

Do we ever return to the original note ?

(i.e. some multiple of an octave)

$$\frac{2}{3} \cdot \cdots \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{2^n}{3^n}$$

Why do we have 12 notes?

That is, can we find m and n such that

$$n \text{ perfect fifths } = \frac{2^n}{3^n} = \frac{1}{2^m} = m \text{ octaves } \textbf{?}$$

Why do we have 12 notes?

That is, can we find m and n such that

$$2^{n+m} = 3^n$$

?

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EVEN

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?

EVEN ODD

Why do we have 12 notes?

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EVEN ODD

NO!

Why do we have 12 notes?

However: We do have $2^{19} = 2^{12+7} \approx 3^{12}$

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12 perfect fifths $\approx$ 7 octaves

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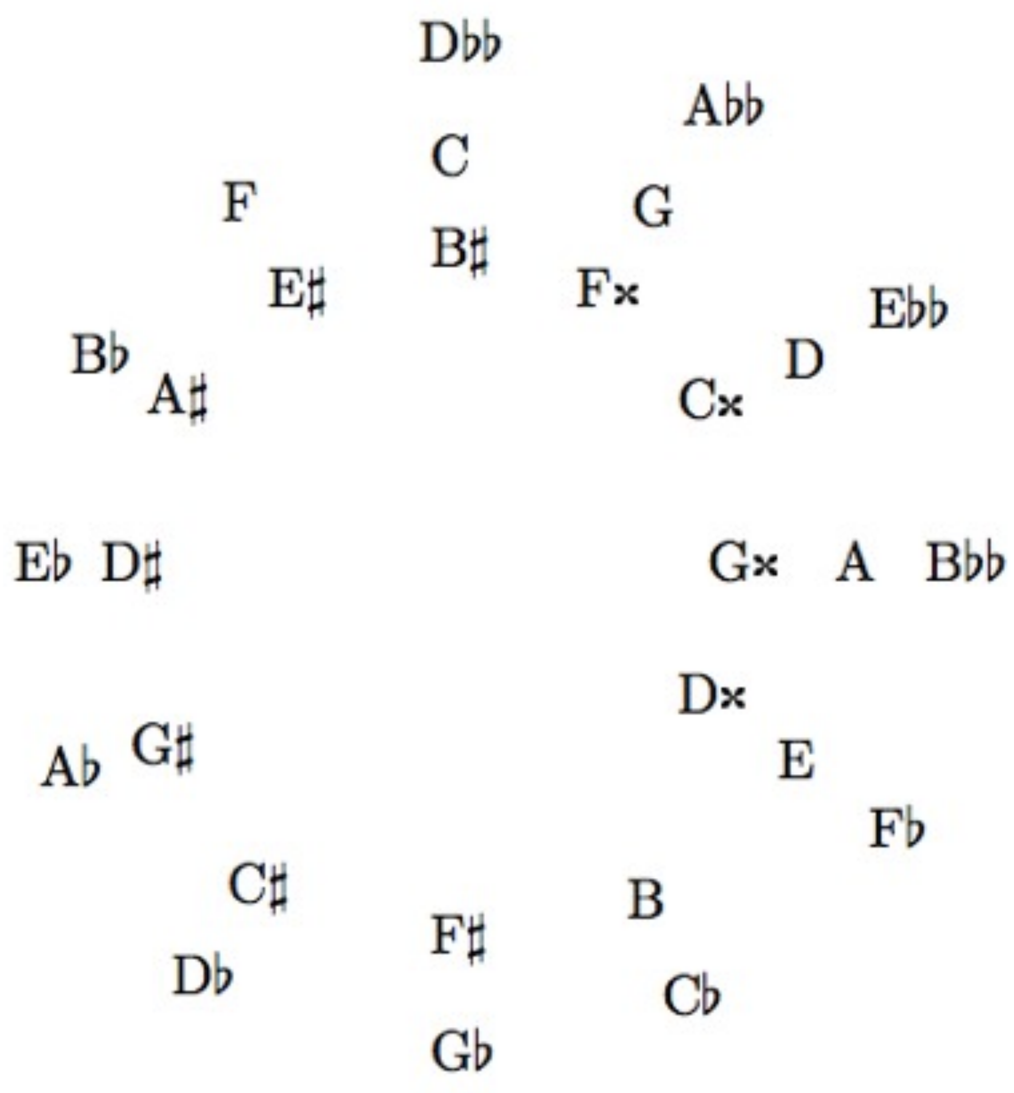
However: We do have $2^{19} = 2^{12+7} \approx 3^{12}$

12 perfect fifths $\approx$ 7 octaves

That’s why we have 12 notes!

Why do we have 12 notes?

But PYTHAGOREAN TUNING has its issues...



the “spiral of fifths”

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These days we use EQUAL TEMPERAMENT TUNING

- divide the octave into 12 EQUAL RATIOS
- It's a trade-off.

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So instead of **2:3**

we use **2:2.9966...**

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These days we use EQUAL TEMPERAMENT TUNING

- divide the octave into 12 EQUAL RATIOS
- It's a trade-off.

So instead of **2:3**

we use **2:2.9966...**

(not exactly "perfect")

Music of the Spheres

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"By the assumption of what uniform and orderly motions can the apparent motions of the planets be accounted for?"

— Plato

Music of the Spheres

Theory:

The moon, planets and stars are fastened to revolving perfect crystalline spheres.

Music of the Spheres

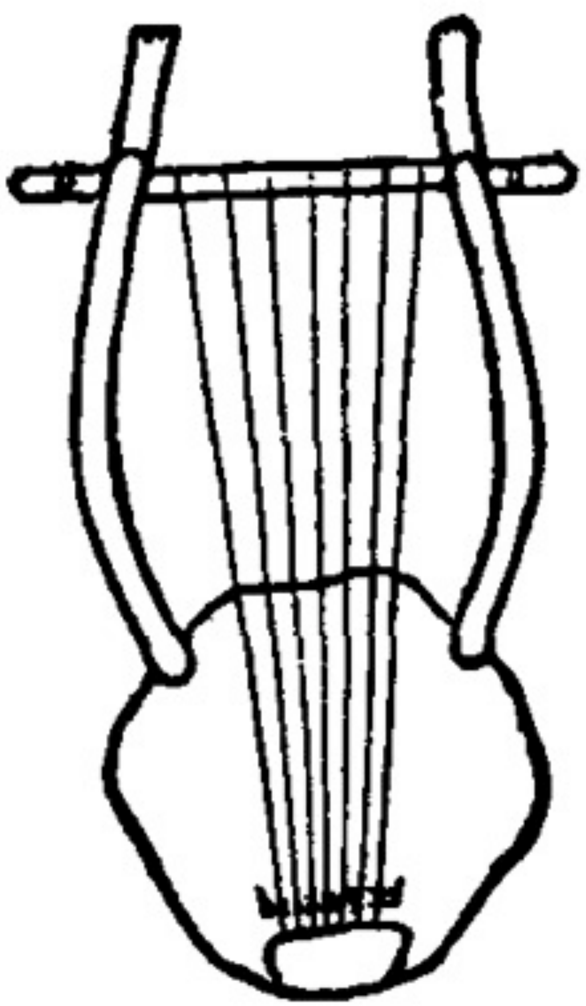
Theory:

The moon, planets and stars are fastened to revolving perfect crystalline spheres.

The radii of the spheres have small whole number ratios.

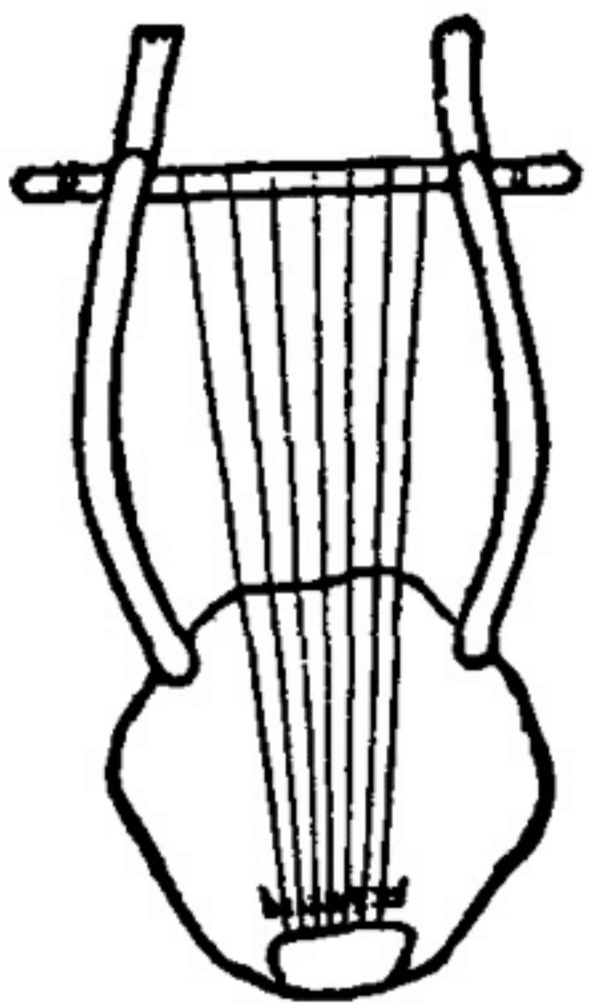
Music of the Spheres

A Grand Synthesis:



Music of the Spheres

Sure, it was “wrong”, but...



Music of the Spheres

was it really so strange?

Music of the Spheres

Thank You!