Noncommutative Algebra **Fall 2016**
Exercises 3 **Drew Armstrong**

3.1. Let $\mathcal{I}$ be a small category and let $\mathcal{C}$ be any category. For each object $c \in \mathcal{C}$ we define the *constant diagram* $\Delta(c) : \mathcal{I} \to \mathcal{C}$ which sends each object of $\mathcal{I}$ to c and each arrow of $\mathcal{I}$ to id_c.

(a) Verify that the map $c \mapsto \Delta(c)$ defines a functor $\Delta : \mathcal{C} \to \mathcal{C}^{\mathcal{I}}$, called the *diagonal functor*.

(b) Consider any diagram $D : \mathcal{I} \to \mathcal{C}$ and object $c \in \mathcal{C}$. Verify that a natural transformation $\alpha : \Delta(c) \Rightarrow D$ is the same thing we previously called a "cone under D".

3.2. State the definition for the *Cartesian product* of categories. For any category $\mathcal{C}$, verify that the assignment $(x, y) \mapsto \mathsf{Hom}_{\mathcal{C}}(x, y)$ defines a functor

$$\mathsf{Hom}_{\mathcal{C}}(-,-) : \mathcal{C}^{\mathsf{op}} \times \mathcal{C} \to \mathsf{Set}.$$

3.3. Verify that the ugly and fancy definitions of adjoint functors in the notes are equivalent.

3.4. Work through the details of the Yoneda Lemma with me.