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1. Difference of Like Powers. Let n be a positive integer and define $\omega := e^{2\pi i/n}$. Prove that for all numbers a and b we have

$$a^n - b^n = (a-b)(a-\omega b)(a-\omega^2 b)\cdots(a-\omega^{n-1}b).$$

2. Roots of Numbers Other Than 1.

(a) Compute the fourth roots of -1.

(b) Use part (a) to factor x^4+1 over the **real numbers**.

3. Cyclotomic Polynomials. We say that $\zeta \in \mathbb{C}$ is a **primitive** nth root of 1 if (1) $\zeta^n = 1$ and (2) $\zeta^m \neq 1$ for $m < n$. The nth cyclotomic polynomial is defined by

$$\Phi_n(x) := \prod_{\zeta}(x-\zeta)$$

where ζ runs over the primitive nth roots of 1.

(a) Find all the primitive 8th roots of 1.

(b) Use part (a) to compute $\Phi_8(x)$.

(c) Use part (b) to completely factor $x^8 - 1$ over the integers.

4. Trisecting an Angle.

(a) Use de Moivre's Theorem to express $\cos(3\theta)$ as a polynomial in $\cos(\theta)$.

(b) Solve the polynomial equation from part (a) to express $\cos(\theta)$ in terms of $\cos(3\theta)$.

(c) Use part (b) to find the exact value of $\cos(\pi/9)$.

5. Rational Root Test. Let $f(x) \in \mathbb{Z}[x]$ be a polynomial with integer coefficients, say $f(x) = c_0 + c_1x + c_2x^2 + \cdots + c_nx^n$ with $c_n \neq 0$.

(a) If $f(a/b) = 0$ for some integers $a, b \in \mathbb{Z}$ with no common factor, prove that a divides c_0 and b divides c_n. [Hint: Multiply both sides of $f(a/b) = 0$ by b^n.]

(b) Use part (a) to prove that the polynomial $f(x) = x^3 - 3x - 1$ has **no rational root**.