Math 310 **Fall 2023**
Homework 1 **Drew Armstrong**

Problem 1. Lines and Circles. The parametrized curve in part (a) is a line. The parametrized curve in part (b) is a circle. In each case, compute the velocity and speed at time t. Also eliminate t to find an equation for the curve in terms of x and y.

(a) $(x,y) = (p+ut, q+vt)$ where p,q,u,v are constants.
(b) $(x,y) = (a+r\cos(\omega t), b+r\sin(\omega t))$ where a,b,r,ω are constants.

Problem 2. Semi-Cubical Parabola. Consider the parametrized curve

$$(x,y) = (t^2, t^3).$$

(a) Eliminate t to find an equation relating x and y. [Hint: Note that $y/x = t$.]
(b) Compute the velocity and speed at time t.
(c) Find the slope of the tangent line at time t.
(d) Use the information in (b) and (c) to sketch the curve for t from $-\infty$ to ∞.

Problem 3. The Cycloid. The cycloid is an interesting curve whose arc length can be computed by hand. It is parametrized by

$$(x,y) = (t-\sin t, 1-\cos t).$$

(a) Check that the slope of the tangent at time t is $\sin t/(1-\cos t)$. Use this information to sketch the curve between $t=0$ and $t=2\pi$. [Hint: The slope goes to infinity when $t\to 0$ from the right and when $t\to 2\pi$ from the left. You do not need to prove this.]
(b) Compute the arc length between $t=0$ and $t=2\pi$. [Hint: You will need the trig identities $\sin^2 t+\cos^2 t = 1$ and $1-\cos t = 2\sin^2(t/2)$.]

Problem 4. A Triangle in the Plane. Consider the following points in $\mathbb{R}^2$:

$$P=(1,3),\quad Q=(-1,2),\quad R=(2,4).$$

(a) Draw the three points together with the midpoints $(P+Q)/2$, $(P+R)/2$, $(Q+R)/2$ and the center of mass $(P+Q+R)/3$.
(b) Find the coordinates of the three side vectors $\mathbf{u}=\vec{PQ}, \mathbf{v}=\vec{QR}, \mathbf{w}=\vec{PR}$.
(c) Use the length formula to compute the three side lengths $\|\mathbf{u}\|, \|\mathbf{v}\|, \|\mathbf{w}\|$.
(d) Use the dot product to compute the three angles of the triangle.

Problem 5. A Triangle in Space. Consider the following points in $\mathbb{R}^3$:

$$P=(1,0,0),\quad Q=(1,1,0),\quad R=(1,1,1).$$

(a) Find the coordinates of the three side vectors $\mathbf{u}=\vec{PQ}, \mathbf{v}=\vec{QR}, \mathbf{w}=\vec{PR}$.
(b) Use the length formula to compute the three side lengths $\|\mathbf{u}\|, \|\mathbf{v}\|, \|\mathbf{w}\|$.
(c) Use the dot product to compute the three angles of the triangle.

Problem 6. Some Vector Arithmetic. Let $\mathbf{u}$ and $\mathbf{v}$ be any two vectors in 100-dimensional space. Use the properties of the dot product to show that

$$\|\mathbf{u}-\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2(\mathbf{u}\bullet\mathbf{v}).$$

[Hint: Start with the definition $\|\mathbf{u}-\mathbf{v}\|^2 = (\mathbf{u}-\mathbf{v})\bullet(\mathbf{u}-\mathbf{v})$, then expand using FOIL.]