Math 309 **Summer B 2018**
Quiz 4 **Drew Armstrong**

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1. Accurately state the Division Theorem.

For all integers $a, b \in \mathbb{Z}$ with $b \neq 0$, there exist unique integers $q, r \in \mathbb{Z}$ satisfying:

$$\begin{cases} a = qb + r, \\ 0 \leq r < |b|. \end{cases}$$

2. Let $a, b \in \mathbb{Z}$ and consider the following statement:

$$\text{“}2|a \Rightarrow 2|(ab).\text{”}$$

(a) Translate the statement into English.

“If a is even then ab is even.”
or
“If 2 divides a then 2 divides ab.”
or
“If there exists $k \in \mathbb{Z}$ such that $a = 2k$, then there exists $\ell \in \mathbb{Z}$ such that $ab = 2\ell$.”

(b) Prove that the statement is true.

Proof: If $2|a$ then by definition we have $a = 2k$ for some $k \in \mathbb{Z}$. But then we also have

$$ab = (2k)b = 2(kb),$$

which by definition says that $2|ab$. □

3. Apply the Euclidean Algorithm to compute greatest common divisor of 105 and 91.

$$\begin{aligned} \mathbf{105} &= 1 \cdot \mathbf{91} + \mathbf{14}, \\ \mathbf{91} &= 6 \cdot \mathbf{14} + \mathbf{7}, \\ \mathbf{14} &= 2 \cdot \mathbf{7} + \mathbf{0}. \end{aligned} \qquad \begin{aligned} \gcd(105, 91) &= \gcd(91, 14) \\ &= \gcd(14, 7) \\ &= \gcd(7, 0) = 7. \end{aligned}$$

4. Apply the Extended Euclidean Algorithm to find the **complete integer solution** $x, y \in \mathbb{Z}$ to the following linear equation:

$$8x + 5y = 1.$$

We make a table of triples $(x, y, z) \in \mathbb{Z}^3$ satisfying $8x + 5y = z$:

x	y	z
1	0	8
0	1	5
1	-1	3
-1	2	2
2	-3	1
-5	8	0

The second-last row gives us one particular solution:

$$8(2) + 5(-3) = 1.$$

And the last row gives us the complete homogeneous solution:

$$8(-5k) + 5(8k) = 0 \qquad \text{for all } k \in \mathbb{Z}.$$

Putting these together gives the complete solution:

$$8(2 - 5k) + 5(-3 + 8k) = 1 \qquad \text{for all } k \in \mathbb{Z}.$$

In other words:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + k \begin{pmatrix} -5 \\ 8 \end{pmatrix} \qquad \text{for all } k \in \mathbb{Z}.$$