Math 309 **Summer B 2018**
Quiz 3 **Drew Armstrong**

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1. Accurately state the Binomial Theorem.

For all integers $n \geq 0$ and for all real numbers x, y we have

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \cdot x^k y^{n-k}$$

2. Draw Pascal's Triangle down the sixth row and use this to find the expansion of $(x+y)^6$.

Here is Pascal's Triangle:

$$\begin{array}{ccccccccccccc} & & & & & & 1 & & & & & & \\ & & & & & 1 & & 1 & & & & & \\ & & & & 1 & & 2 & & 1 & & & & \\ & & & 1 & & 3 & & 3 & & 1 & & & \\ & & 1 & & 4 & & 6 & & 4 & & 1 & & \\ & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\ 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \end{array}$$

Therefore we have

$$(x+y)^6 = 1x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + 1y^6.$$

3. Consider the set $S = \{1,2,3,4,5,6\}$.

(a) How many subsets does S have?

The number of subsets is $2^{\#S} = 2^6 = 64$. Alternatively, we can use Pascal's Triangle:

$$\sum_{k=0}^{6} \binom{6}{k} = 1 + 6 + 15 + 20 + 15 + 6 + 1 = 64.$$

(b) How many of these subsets contain an **even** number of elements? [Note: 0 is even.]

You may remember from class that the number of even subsets is $2^{\#S-1} = 2^5 = 32$. Alternatively, we can use Pascal's Triangle:

$$\binom{6}{0} + \binom{6}{2} + \binom{6}{4} + \binom{6}{6} = 1 + 15 + 15 + 1 = 32.$$

4.

(a) How many words can be made from k copies of "a" and $n-k$ copies of "b"?

This is the well-known binomial coefficient:

$$\frac{n!}{k!(n-k)!}.$$

(b) How many ways are there to arrange the letters "$t, e, n, n, e, s, s, e, e$" ?

There are 9 letters in total, in which

"t" appears 1 time,
"e" appears 4 times,
"n" appears 2 times, and
"s" appears 2 times.

Therefore the number of arrangements is the following multinomial coefficient:

$$\frac{9!}{1!4!2!2!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{2 \cdot 2} = 3,780.$$

Since there's extra white space, here's a free remark:

$$(t+e+n+s)^9 = \cdots + 3780 \cdot t^1 e^4 n^2 s^2 + \cdots .$$