

1. Let n be a positive whole number. Find a **closed form** for the following sum:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n$$

2. Let n be a positive whole number. Find a **closed form** for the following sum:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

3. Translate the following statement into English. (Many correct answers.)

$$\forall x \in S, P(x)$$

“**Every** element x of the set S satisfies property $P(x)$.”

4. Translate the following statement into English. (Many correct answers.)

$$\exists x \in S, \neg P(x)$$

“There **exists** an element x of the set S such that property $P(x)$ does **not** hold.”

5. What is the logical relationship between the statements in Problems 3 and 4?

They are opposites. Writing symbolically, we have

$$\neg(\forall x \in S, P(x)) \equiv \exists x \in S, \neg P(x).$$