Math 309 **Summer 2014**
Homework 5 **Drew Armstrong**

The purpose of this homework is to let you practice the technique of induction. Each proof should take up a good amount of space. Don't try to shrink it down to one paragraph. Make sure that the logical structure of the proof is expressed clearly.

1. Let r be a real number other than 1. Use induction to prove that for all $n \in \mathbb{N}$ we have

$$1 + r + r^2 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}.$$

2. Use induction to prove that $n^3 + 3n^2 + 2n$ is divisible by 6 for all $n \in \mathbb{N}$. [Recall: We say that $m \in \mathbb{Z}$ is divisible by 6 if there exists an integer $d \in \mathbb{Z}$ such that $m = 6d$.]

3. Let n be any positive integer and let $A_1, A_2, \ldots, A_n$ be any n subsets of the universal set U. Use induction to prove that

$$(A_1 \cup A_2 \cup \cdots \cup A_n)^c = A_1^c \cap A_2^c \cap \cdots \cap A_n^c.$$

[Hint: Let $P(n) =$"Given **any** n subsets $A_1, A_2, \ldots, A_n$ of the universal set U, we have $(A_1 \cup \cdots \cup A_n)^c = A_1^c \cap \cdots \cap A_n^c$". I know it's long, but this is the $P(n)$ we want to prove. Don't shorten it! The base case is $n = 2$. How do we know that $P(2)$ is a true statement?]

4. Use induction to prove that there are 2^n binary strings of length n for all $n \in \mathbb{N}$. [Hint: Let $P(n) =$"There are 2^n binary strings of length n". For the induction step, let $k \geq 0$ and assume that $P(k) = T$. In this (hypothetical) case you want to show that $P(k+1) = T$. To do this, let S be the set of binary strings of length $k+1$. Write $S = A \sqcup B$, where A is the subset of binary strings that begin with 0, and B is the subset of binary strings that begin with 1. Explain why you know that that $\#A = \#B = 2^k$. Now what?]