Math 309 **Summer 2014**
Homework 1 **Drew Armstrong**

Let L_n be the maximum number of regions we can get by drawing n (infinite) lines in the plane. We showed in class that

$$L_n = 1 + (1 + 2 + 3 + \cdots + n) = 1 + \sum_{k=1}^{n} k = 1 + \frac{n(n+1)}{2} = \frac{n^2 + n + 2}{2}.$$

1. Let f and g be two functions of a discrete variable n. We write $f(n) \sim g(n)$ (and we say that $f(n)$ is asymptotic to $g(n)$) if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1.$$

Show that $L_n \sim \frac{1}{2}n^2$.

2. We proved in class that

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Use this formula to evaluate the sum $\sum_{k=1}^{n}(a + bk + ck^2)$, where a, b, c are arbitrary constants.

3. Now let P_n be the maximum number of 3-dimensional regions we can get by cutting 3-dimensional space with n infinite planes (i.e., the maximum number of pieces of cheese we can get using n cuts). You can assume that for all $n > 0$ we have

$$P_{n+1} = P_n + L_n.$$

(a) Use this recurrence to show that for all $n > 0$ we have

$$P_n = 1 + L_0 + L_1 + L_2 + \cdots L_{n-1} = 1 + \sum_{k=0}^{n-1} L_k.$$

(b) Use the result of part (a) to show that for all $n > 0$ we have

$$P_n = \frac{n^3 + 5n + 6}{6}.$$

[Hint: Use Problem 2.] It just so happens that this formula also works when $n = 0$.