Math 230 **Spring 2019**
Homework 2 **Drew Armstrong**

Problem 1. Let P and Q be any mathematical statements.

(a) Use a truth table to prove **de Morgan's Laws**:
$$\neg(P \vee Q) = (\neg P \wedge \neg Q) \qquad \text{and} \qquad \neg(P \wedge Q) = (\neg P \vee \neg Q).$$
(b) Use a truth table to verify that $(P \Rightarrow Q) = (\neg P \vee Q)$.

(c) Use part (b) to prove the **contrapositive principle**:
$$(P \Rightarrow Q) = (\neg Q \Rightarrow \neg P).$$
Do **not** use a truth table.

Problem 2. Let P, Q, R be any mathematical statements.

(a) Use parts (a) and (b) of Problem 1 to verify that
$$P \Rightarrow (Q \vee R) = (P \wedge \neg Q) \Rightarrow R.$$
Again, do **not** use a truth table. [Hint: You can assume that $\neg(\neg A) = A$ and $A \vee (B \vee C) = (A \vee B) \vee C$ for any statements A, B, C.]

(b) Use the logical principle from part (a) to prove that the following silly statement is true for all integers $m, n \in \mathbb{Z}$:

" If m is odd, then either n is even or mn is odd (or both). "

[Hint: What are the statements P, Q, R in this case?]

Problem 3. In this problem you will prove that $\sqrt{5}$ is irrational.

(a) There are four different ways that an integer can be "not a multiple of 5." List them.

(b) Use part (a) and the contrapositive to prove for all integers n that
$$(n^2 \text{ is a multiple of } 5) \Longrightarrow (n \text{ is a multiple of } 5.)$$
[Hint: This will be a case-by-case proof.]

(c) Use part (b) and proof by contradiction to show that $\sqrt{5}$ is not a fraction of whole numbers. [Hint: Try to mimic the proof from class as closely as possible.]

Problem 4. For any integer $n \in \mathbb{Z}$ consider the following mathematical statement:
$$P(n) := \text{“}1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1).\text{”}$$
(a) Verify that the statements $P(1)$, $P(2)$ and $P(3)$ are all true.

(b) Now fix an arbitrary positive integer $k \geq 1$ and **assume for induction** that the statement $P(k)$ is true. In this case prove that the statement $P(k+1)$ is also true.

(c) What do you conclude from this?