Problem 1. Proposition I.5 in Euclid has acquired the name *pons asinorum*, which translates as “bridge of asses” or “bridge of fools”. Apparently, many students never got past this proposition. (I would say that the *pons asinorum* in today’s curriculum is addition of fractions.) The proposition says the following: Consider a triangle $\triangle ABC$. If the side lengths $AB$ and $AC$ are equal (i.e. the triangle is *isosceles*), then the angles $\angle ABC$ and $\angle ACB$ are equal.

Your assignment is to look up Euclid’s proof and tell it to me.

Problem 2. Prove that the interior angles of any triangle sum to $180^\circ$. You may use the following two facts without justification. **Fact 1:** Given a line $\ell$ and a point $p$ not on $\ell$, it is possible to draw a line through $p$ parallel to $\ell$. **Fact 2:** If a line falls on two parallel lines, then the corresponding angles are equal, as in the following figure.

\[ \begin{array}{c}
\alpha \\
\beta
\end{array} \]

Problem 3. Prove Thales’ Theorem, which says the following. Consider a triangle $\triangle ABC$ inscribed in a circle. If line segment $BC$ is a diameter of the circle, then angle $\angle BAC$ is a right angle. You may quote the results from Problems 1 and 2.
Problem 4. The dot product of vectors $u = (u_1, u_2, \ldots, u_n)$ and $v = (v_1, v_2, \ldots, v_n)$ is defined by $u \cdot v := u_1v_1 + u_2v_2 + \cdots + u_nv_n$. The length $\|u\|$ of a vector $u$ is defined by $\|u\|^2 := u \cdot u$.

(a) Prove the formula $\|u - v\|^2 = \|u\|^2 + \|v\|^2 - 2(u \cdot v)$.

(b) Use this formula together with the 2D Pythagorean Theorem and its converse to prove the following statement:

"the vectors $u$ and $v$ are perpendicular if and only if $u \cdot v = 0$.”

[Hint: Where is the triangle? Recall that you must prove both directions of the if and only if statement separately.]

Problem 5. Use vectors to give an analytic proof of Thales’ Theorem. [Hint: You may assume that your circle is the unit circle in the Cartesian plane. You may assume that $B = (-1, 0)$, $C = (1, 0)$, and $A = (\cos \theta, \sin \theta)$ for some angle $\theta$. Consider the vectors $u = A - B$ and $v = A - C$. Compute the dot product $u \cdot v$.]