Math 224 **Fall 2017**
Homework 4 **Drew Armstrong**

Problems from 9th edition of *Probability and Statistical Inference* by Hogg, Tanis and Zimmerman:

- Section 2.3, Exercises 16(a,d),18.
- Section 2.4, Exercises 13, 14.
- Section 4.1, Exercises 3, 4.
- Section 4.2, Exercises 3(a).
- Section 5.3, Exercises 2, 5.

Additional Problems.

1. "Collecting Coupons." Each box of a certain brand of cereal comes with a toy. If there are n possible toys and if they are distributed randomly, how many boxes of cereal do you expect to buy before you get them all?

(a) Let X be a geometric random variable with pmf $P(X = k) = p(1-p)^{k-1}$. Use a geometric series to compute the moment generating function:

$$M(t) = E[e^{tX}] = \sum_{k=1}^{\infty} e^{tk} p(1-p)^{k-1} = e^t p \cdot \sum_{k=1}^{\infty} \left[e^t(1-p)\right]^{k-1} = ?$$

(b) Compute the derivative of $M(t)$ to find the expected value of X:

$$E[X] = M'(0) = ?$$

(b) Assuming that you already have ℓ of the toys, let X_ℓ be the number of boxes of cereal that you buy until you get a new toy. Observe that X_ℓ is geometric and use this fact to compute $E[X_\ell]$.

(d) Let X be the number of boxes that you buy until you see all n toys. Then we have

$$X = X_0 + X_1 + \cdots + X_{n-1}.$$

Use this to compute the expected value $E[X]$. [Hint: See Example 2.5-5 in the textbook for the case $n = 6$.]