Math 210 F — Spring 2016
Homework 3 — Drew Armstrong

Problem 1. In class I stated that a system of **linear** equations has either 0, 1, or ∞ many solutions. Let's examine this claim.

(a) Suppose that (x_1, y_1, z_1) and (x_2, y_2, z_2) are two solutions to the linear equation

$$ax + by + cz = d.$$

In this case, show that the midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$ is also a solution.

(b) Fill in the blank: If 25 hyperplanes in 12-dimensional space meet at two given points, they they must also meet at ______ .

Problem 2. Consider the following linear system:

$$\begin{cases} x + y + z = 2 \\ x + 2y + z = 3 \\ x + 3y + 2z = 5 \end{cases}$$

(a) Compute the RREF of the system.
(b) Describe the row picture of the solution.
(c) Describe the column picture of the solution.

Problem 3. Now consider the modified system:

$$\begin{cases} x + y + z = 2 \\ x + 2y + z = 3 \\ 2x + 3y + 2z = c \end{cases}$$

where c is an arbitrary constant.

(a) Put the system in staircase form. You don't need to compute the RREF.
(b) Fill in the blanks: The first two planes meet in a line L. When $c = 5$ we have ∞ many solutions because the third plane ______ , but when $c = 6$ we have 0 solutions because the third plane ______ .
(c) Fill in the blank: It is impossible for the system to have exactly 1 solution because if we have one solution

$$x_1 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + y_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + z_1 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ c \end{pmatrix},$$

then we also have another solution ______ . [Hint: Change x_1 and z_1 somehow. The value of c is irrelevant.]

Problem 4. Consider the following linear system:

$$\begin{cases} 0 + x_2 + 0 + x_4 - x_5 - 4x_6 = -1 \\ x_1 + 2x_2 - x_3 + 4x_4 - x_5 - 4x_6 = 3 \\ x_1 + 2x_2 - x_3 + 4x_4 + 0 - x_6 = 5 \end{cases}$$

(a) Compute the RREF of the system.
(b) Write down the full solution in parametric form.