Math 210 E **Spring 2013**

Homework 7 **Drew Armstrong**

Reading:

Section 4.3

Problems:

Section 4.3: 5, 7, 12, 17, 22

Additional Problems:

For these problems, use the following definition:

> We say that P is a **projection matrix** if $P^T = P$ (that is, P is "symmetric") and $P^2 = P$ (that is, P is "idempotent").

A.1. If A is a rectangular matrix such that $(A^TA)^{-1}$ exists, show that $P = A(A^TA)^{-1}A^T$ is a projection matrix.

A.2. If A is a square invertible matrix, show that $P = A(A^TA)^{-1}A^T = I$. What does this mean? What subspace does P project onto?

A.3. If P is a projection, show that $I - P$ is also a projection.

A.4. Show that the projections P and $I - P$ satisfy $P(I - P) = 0$ (the zero matrix). We say that the projections P and $I - P$ are are **orthogonal** to each other, because they project onto orthogonal subspaces.

Hint for the Additional Problems: The following identities hold whenever the products and inverses under discussion exist.

$$\begin{aligned}(AB)^{-1} &= B^{-1}A^{-1}\\ (AB)^T &= B^TA^T\\ (A^T)^{-1} &= (A^{-1})^T.\end{aligned}$$