Math 161 **Summer 2023**
Homework 1 **Drew Armstrong**

Book Problems:

- Section 1.3, Exercise 13
- Section 1.4, Exercises 12, 18, 22, 28, 50, 52, 54, 56
- Section 1.6, Exercises 14, 18, 24, 28, 30

Additional Problems:

A1. The Geometric Series. For any real number r and for any positive integer n we have the following identity:

$$1 - r^{n+1} = (1-r)(1 + r + r^2 + \cdots + r^n).$$

(a) If $|r| < 1$, use this identity to prove that

$$\lim_{n\to\infty}(1 + r + r^2 + \cdots + r^n) = \frac{1}{1-r}.$$

[Hint: If $|r| < 1$ then we have $\lim_{n\to\infty} r^n = 0$.]

(b) Use part (a) to evaluate the infinite sum

$$1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots.$$

A2. The Exponential Function. Recall that the number e is defined by the limit

$$e := \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n.$$

In class we interpreted this as the amount of money you will have after one year if you invest \$1 in a bank account with 100% yearly rate of return. Using the same reasoning, if you invest \$1 in a bank account with yearly rate of return $r > 0$ (the rate $r = 1$ corresponds to 100%), then the amount of money you will have after one year is

$$\lim_{n\to\infty}\left(1 + \frac{r}{n}\right)^n.$$

Use the **substitution method** to evaluate this limit. [Hint: Let $n = mr$ and note that $m \to \infty$ as $n \to \infty$.]