Math 161 **Summer 2015**
Quiz 4 **Drew Armstrong**

Each problem is worth 2 points.

1. Compute the most general antiderivative of $f(x) = x^2 - 2x + 3$.

Using the power rule $\int x^p\,dx = \frac{1}{p+1}x^{p+1}$ (when $p \neq -1$) gives

$$\int f(x)\,dx = \int x^2\,dx - 2\int x\,dx + 3\int 1\,dx = \frac{1}{3}x^3 - 2\frac{1}{2}x^2 + 3x + C,$$

where C is an arbitrary constant.

2. Suppose that $s''(t) = -10$ for all t. Compute $s(t)$ assuming that $s'(0) = 2$ and $s(0) = 5$.

First we compute

$$s'(t) = \int s''(t)\,dt = \int -10\,dt = -10t + C.$$

Since $2 = s'(0) = -10(0) + C = C$ we conclude that

$$s'(t) = -10t + 2.$$

Then we compute

$$s(t) = \int s'(t)\,dt = \int (-10t + 2)\,dt = -10\frac{1}{2}t^2 + 2t + D = -5t^2 + 2t + D.$$

Since $5 = s(0) = -5(0)^2 + 2(0) + D = D$ we conclude that

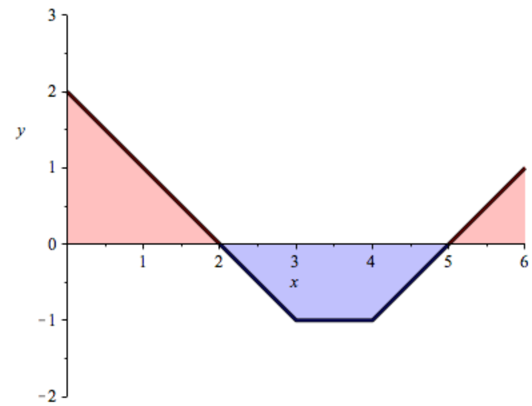
$$s(t) = -5t^2 + 2t + 5.$$

3. Let c be constant. Evaluate the integral $\displaystyle\int_a^b c\,dx$.

There are two ways to do this. First, we could note that the graph of $f(x) = c$ is a horizontal line at height c. The region between this and the x-axis from $x = a$ to $x = b$ is a rectangle with width $b - a$ and height c, so it has area $c(b - a)$. On the other hand, we could note that $F(x) = cx$ is an antiderivative of $f(x) = c$ and then use the F.T.C.:

$$\int_a^b c\,dx = F(b) - F(a) = cb - ca = c(b - a).$$

4. The following picture shows the graph of $g(x)$. Use this to compute $\int_0^6 g(x)\,dx$.



The integral computes the area above the graph (in pink) **minus** the area below the graph (in blue). The area above the graph is two triangles with areas $2 \cdot 2/2 = 2$ and $1 \cdot 1/2 = 1/2$. The area below the graph is a square and two triangles, so it has area $1 \cdot 1/2 + 1 \cdot 1 + 1 \cdot 1/2 = 2$. Therefore,

$$\int_0^6 g(x)\,dx = 2 - 2 + 1/2 = 1/2.$$

5. Evaluate the integral $\int_1^2 \sqrt{t}\,(1+t)\,dt$.

Let $f(t) = \sqrt{t}(1+t)$. First we expand to get $f(t) = \sqrt{t} + \sqrt{t}t = t^{1/2} + t^{3/2}$. One particular antiderivative of this is

$$F(t) = \frac{t^{3/2}}{3/2} + \frac{t^{5/2}}{5/2} = \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2}.$$

Then the F.T.C. gives

$$\begin{aligned}
\int_1^2 f(t) &= F(2) - F(1) \\
&= \left(\frac{2}{3}(2)^{3/2} + \frac{2}{5}(2)^{5/2}\right) - \left(\frac{2}{3}(1)^{3/2} + \frac{2}{5}(1)^{5/2}\right) \\
&= \left(\frac{2}{3} \cdot 2\sqrt{2} + \frac{2}{5} \cdot 2^2\sqrt{2}\right) - \left(\frac{2}{3} + \frac{2}{5}\right) \\
&= \left(\frac{4}{3}\sqrt{2} + \frac{8}{5}\sqrt{2}\right) - \left(\frac{2}{3} + \frac{2}{5}\right) \\
&= \frac{44}{15}\sqrt{2} - \frac{16}{15}.
\end{aligned}$$

[Remark: You did not need to simplify.]